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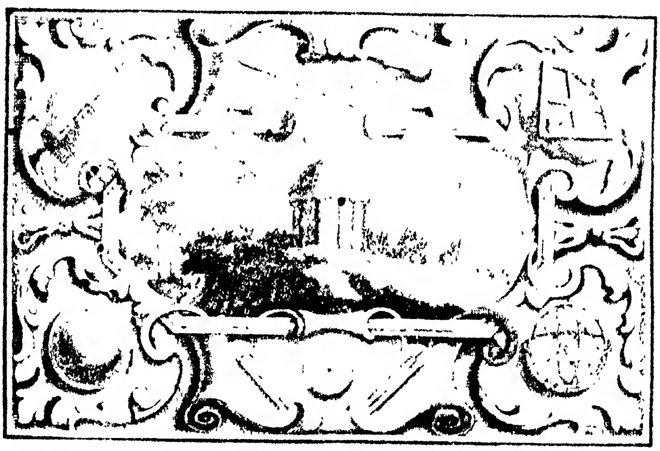
VIEW

OF

Sir *ISAAC NEWTON's*

PHILOSOPHY



LONDON:

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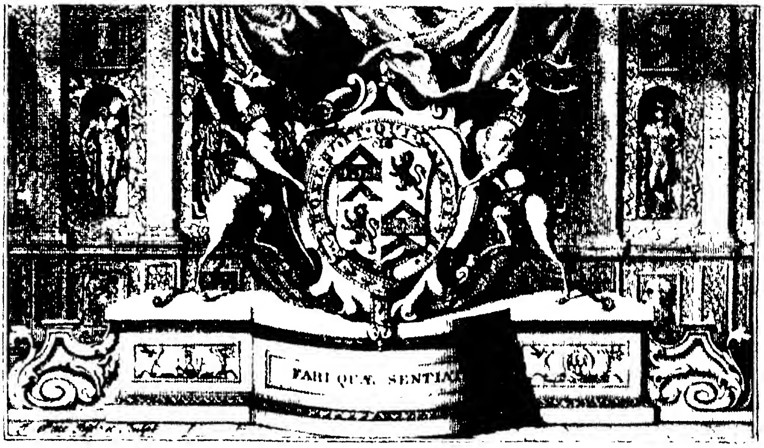

To the Noble and Right Honourable

Sir *ROBERT WALPOLE*.

SIR,

I Take the liberty to ſend you this view of Sir Isaac Newton's philoſophy, which, if it were performed ſuitable to the dignity of the ſubject, might not be a preſent unworthy the acceptance of the greateſt perſon. For his philoſophy affords us the only true account of the

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operations of nature, which for ſo many ages had imployed the curioſity of mankind; though no one before him was furniſhed with the ſtrength of mind neceſſary to go any depth in this difficult ſearch. However, I am encouraged to hope, that this attempt, imperfect as it is, to give our countrymen in general ſome conception of the labours of a perſon, who ſhall always be the boaſt of this nation, may be received with indulgence by one, under whoſe influence theſe kingdoms enjoy ſo much happineſs. Indeed my admiration at the ſurprizing inventions of this great man, carries me to conceive of him as a perſon, who not only muſt raiſe the glory of the country, which gave him birth; but that he has even done honour to human nature, by having extended the greateſt and moſt noble of our faculties, reaſon, to ſubjects, which, till he attempted them, appeared to be wholly beyond the reach of our limited capacities. And what can give us a

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more pleasing prospect of our own condition, than to see so exalted a proof of the strength of that faculty, whereon the conduct of our lives, and our happiness depends; our passions and all our motives to action being in such manner guided by our opinions, that where these are just, our whole behaviour will be praise-worthy? But why do I presume to detain you, SIR, with such reflections as these, who must have the fullest experience within your own mind, of the effects of right reason? For to what other source can be ascribed that amiable frankness and unreserved condescension among your friends, or that masculine perspicuity and strength of argument, whereby you draw the admiration of the publick, while you are engaged in the most important of all causes, the liberties of mankind?

I humbly crave leave to make the only acknowledgement within my power, for the benefits, which

which I receive in common with the reſt of my countrymen from theſe high talents, by ſubſcribing my ſelf

SIR,

Your moſt faithful,

and

Moſt humble Servant,

HENRY PEMBERTON.

PREFACE.

I Drew up the following papers many years ago at the desire of some friends, who, upon my taking care of the late edition of Sir ISAAC NEWTON'*s Principia, perswaded me to make them publick. I laid hold of that opportunity, when my thoughts were afresh employed on this subject, to revise what I had formerly written. And I now send it abroad not without some hopes of answering these two ends. My first intention was to convey to such, as are not used to mathematical reasoning, some idea of the philosophy of a person, who has acquired an universal reputation, and rendered our nation famous for these speculations in the learned world. To which purpose I have avoided using terms of art as much as possible, and taken care to define such as I was obliged to use. Though this caution was the less necessary at present, since many of them are become familiar words to our language, from the great number of books wrote in it upon philosophical subjects, and the courses of experiments, that have of late years been given by several ingenious men. The other view I had, was to encourage such young gentlemen as have a turn for the mathematical sciences, to pursue those studies the more chearfully, in order to understand in our author himself the demonstrations of the things I here declare. And to facilitate their progress herein, I intend to proceed still farther in the explanation of Sir* ISAAC NEWTON'*s philosophy. For as I have received very much pleasure from perusing his writings, I hope it is no illaudable ambition to endeavour the rendering them more easily understood, that greater numbers may enjoy the same satisfaction.*

It will perhaps be expected, that I should say something particular of a person, to whom I must always acknowledge my self to be much obliged. What I have to declare on this head will be but short; for it was in the very last years of Sir ISAAC'*s life, that I had the ho-*

nour of his acquaintance. This happened on the following occasion. *Mr.* Polenus, *a Professor in the University of* Padua, *from a new experiment of his, thought the common opinion about the force of moving bodies was overturned, and the truth of Mr.* Libnitz's *notion in that matter fully proved. The contrary of what* Polenus *had asserted I demonstrated in a paper, which Dr.* MEAD, *who takes all opportunities of obliging his friends, was pleased to shew Sir* ISAAC NEWTON. *This was so well approved of by him, that he did me the honour to become a fellow-writer with me, by annexing to what I had written, a demonstration of his own drawn from another consideration. When I printed my discourse in the philosophical transactions, I put what Sir* ISAAC *had written in a scholium by it self, that I might not seem to usurp what did not belong to me. But I concealed his name, not being then sufficiently acquainted with him to ask whether he was willing I might make use of it or not. In a little time after he engaged me to take care of the new edition he was about making of his Principia. This obliged me to be very frequently with him, and as he lived at some distance from me, a great number of letters passed between us on this account. When I had the honour of his conversation, I endeavoured to learn his thoughts upon mathematical subjects, and something historical concerning his inventions, that I had not been before acquainted with. I found, he had read fewer of the modern mathematicians, than one could have expected; but his own prodigious invention readily supplied him with what he might have an occasion for in the pursuit of any subject he undertook. I have often heard him censure the handling geometrical subjects by algebraic calculations; and his book of Algebra he called by the name of Universal Arithmetic, in opposition to the injudicious title of Geometry, which* Des Cartes *had given to the treatise, wherein he shews, how the geometer may assist his invention by such kind of computations. He frequently praised* Slusius, Barrow *and* Huygens *for not being influenced by the false taste, which then began to prevail. He used to commend the laudable attempt of* Hugo de Omerique *to restore the ancient analysis, and very much esteemed* Apollonius's *book* De sectione rationis *for giving us a clearer notion of that analysis than we had before. Dr.* Barrow *may be esteemed as hav-*

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ing shewn a compass of invention equal, if not superior to any of the moderns, our author only excepted; but Sir ISAAC NEWTON *has several times particularly recommended to me* Huygens's *stile and manner. He thought him the most elegant of any mathematical writer of modern times, and the most just imitator of the antients. Of their taste, and form of demonstration Sir* ISAAC *always professed himself a great admirer. I have heard him even censure himself for not following them yet more closely than he did; and speak with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of* Des Cartes *and other algebraic writers, before he had considered the elements of* Euclide *with that attention, which so excellent a writer deserves. As to the history of his inventions, what relates to his discoveries of the methods of series and fluxions, and of his theory of light and colours, the world has been sufficiently informed of already. The first thoughts, which gave rise to his Principia, he had, when he retired from* Cambridge *in* 1666 *on account of the plague. As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not found sensibly diminished at the remotest distance from the center of the earth, to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains; it appeared to him reasonable to conclude, that this power must extend much farther than was usually thought; why not as high as the moon, said he to himself? and if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby. However, though the power of gravity is not sensibly weakened in the little change of distance, at which we can place our selves from the center of the earth; yet it is very possible, that so high as the moon this power may differ much in strength from what it is here. To make an estimate, what might be the degree of this diminution, he considered with himself, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried round the sun by the like power. And by comparing the periods of the several planets with their distances from the sun, he found, that if any power like gravity held them in their courses, its strength must decrease in the duplicate proportion of the increase of distance. This*

he concluded by supposing them to move in perfect circles concentrical to the sun, from which the orbits of the greatest part of them do not much differ. Supposing therefore the power of gravity, when extended to the moon, to decrease in the same manner, he computed whether that force would be sufficient to keep the moon in her orbit. In this computation, being absent from books, he took the common estimate in use among geographers and our seamen, before Norwood *had measured the earth, that 60 English miles were contained in one degree of latitude on the surface of the earth. But as this is a very faulty supposition, each degree containing about 69½ of our miles, his computation did not answer expectation; whence he concluded, that some other cause must at least join with the action of the power of gravity on the moon. On this account he laid aside for that time any farther thoughts upon this matter. But some years after, a letter which he received from Dr.* Hook, *put him on inquiring what was the real figure, in which a body let fall from any high place descends, taking the motion of the earth round its axis into consideration. Such a body, having the same motion, which by the revolution of the earth the place has whence it falls, is to be considered as projected forward and at the same time drawn down to the center of the earth. This gave occasion to his resuming his former thoughts concerning the moon; and* Picart *in* France *having lately measured the earth, by using his measures the moon appeared to be kept in her orbit purely by the power of gravity; and consequently, that this power decreases as you recede from the center of the earth in the manner our author had formerly conjectured. Upon this principle he found the line described by a falling body to be an ellipsis, the center of the earth being one focus. And the primary planets moving in such orbits round the sun, he had the satisfaction to see, that this inquiry, which he had undertaken merely out of curiosity, could be applied to the greatest purposes. Hereupon he composed near a dozen propositions relating to the motion of the primary planets about the sun. Several years after this, some discourse he had with Dr.* Halley, *who at* Cambridge *made him a visit, engaged Sir* ISAAC NEWTON *to resume again the consideration of this subject; and gave occasion*

to his writing the treatise which he published under the title of mathematical principles of natural philosophy. This treatise, full of such a variety of profound inventions, was composed by him from scarce any other materials than the few propositions before mentioned, in the space of one year and an half.

Though his memory was much decayed, I found he perfectly understood his own writings, contrary to what I had frequently heard in discourse from many persons. This opinion of theirs might arise perhaps from his not being always ready at speaking on these subjects, when it might be expected he should. But as to this, it may be observed, that great genius's are frequently liable to be absent, not only in relation to common life, but with regard to some of the parts of science they are the best informed of. Inventors seem to treasure up in their minds, what they have found out, after another manner than those do the same things, who have not this inventive faculty. The former, when they have occasion to produce their knowledge, are in some measure obliged immediately to investigate part of what they want. For this they are not equally fit at all times: so it has often happened, that such as retain things chiefly by means of a very strong memory, have appeared off hand more expert than the discoverers themselves.

As to the moral endowments of his mind, they were as much to be admired as his other talents. But this is a field I leave others to exspatiate in. I only touch upon what I experienced my self during the few years I was happy in his friendship. But this I immediately discovered in him, which at once both surprized and charmed me: Neither his extreme great age, nor his universal reputation had rendred him stiff in opinion, or in any degree elated. Of this I had occasion to have almost daily experience. The Remarks I continually sent him by letters on his Principia were received with the utmost goodness. These were so far from being any ways displeasing to him, that on the contrary it occasioned him to speak many kind things of me to my friends, and to honour me with a publick testimony of his good opinion. He also approved of the following treatise, a great part of which we read together. As many alterations were

 made

From black obscurity's abyss to raise,
(Drooping and mourning o'er thy wondrous works)
With vain inquiry sought. Like meteors these
In their dark age bright sons of wisdom shone:
But at thy NEWTON all their laurels fade,
They shrink from all the honours of their names.
So glimm'ring stars contract their feeble rays,
When the swift lustre of AURORA's face
Flows o'er the skies, and wraps the heav'ns in light.

THE Deity's omnipotence, the cause,
Th' original of things long lay unknown.
Alone the beauties prominent to sight
(Of the celestial power the outward form)
Drew praise and wonder from the gazing world.
As when the deluge overspread the earth,
Whilst yet the mountains only rear'd their heads
Above the surface of the wild expanse,
Whelm'd deep below the great foundations lay,
Till some kind angel at heav'n's high command
Roul'd back the rising tides, and haughty floods,
And to the ocean thunder'd out his voice:
Quick all the swelling and imperious waves,
The foaming billows and obscuring surge,
Back to their channels and their ancient seats
Recoil affrighted: from the darksome main
Earth raises smiling, as new-born, her head,
And with fresh charms her lovely face arrays.
So his extensive thought accomplish'd first
The mighty task to drive th' obstructing mists
Of ignorance away, beneath whose gloom
Th' inshrouded majesty of Nature lay.
He drew the veil and swell'd the spreading scene.
How had the moon around th' ethereal void

Rang'd,

Rang'd, and eluded lab'ring mortals care,
Till his invention trac'd her ſecret ſteps,
While ſhe inconſtant with unſteady rein
Through endleſs mazes and meanders guides
In its unequal courſe her changing carr:
Whether behind the ſun's ſuperior light
She hides the beauties of her radiant face,
Or, when conſpicuous, ſmiles upon mankind,
Unveiling all her night-rejoicing charms.
When thus the ſilver-treſſed moon diſpels
The frowning horrors from the brow of night,
And with her ſplendors chears the ſullen gloom,
While ſable-mantled darkneſs with his veil
The viſage of the fair horizon ſhades,
And over nature ſpreads his raven wings;
Let me upon ſome unfrequented green
While ſleep ſits heavy on the drowſy world,
Seek out ſome ſolitary peaceful cell,
Where darkſome woods around their gloomy brows
Bow low, and ev'ry hill's protended ſhade
Obſcures the dusky vale, there ſilent dwell,
Where contemplation holds its ſtill abode,
There trace the wide and pathleſs void of heav'n,
And count the ſtars that ſparkle on its robe.
Or elſe in fancy's wild'ring mazes loſt
Upon the verdure ſee the fairy elves
Dance o'er their magick circles, or behold,
In thought enraptur'd with the ancient bards,
Medea's baleful incantations draw
Down from her orb the paly queen of night.
But chiefly NEWTON let me ſoar with thee,
And while ſurveying all yon ſtarry vault
With admiration I attentive gaze,
Thou ſhalt deſcend from thy celeſtial ſeat,

And waft aloft my high-aspiring mind,
Shalt shew me there how nature has ordain'd
Her fundamental laws, shalt lead my thought
Through all the wand'rings of th' uncertain moon,
And teach me all her operating powers.
She and the sun with influence conjoint
Wield the huge axle of the whirling earth,
And from their just direction turn the poles,
Slow urging on the progress of the years.
The constellations seem to leave their seats,
And o'er the skies with solemn pace to move.
You, splendid rulers of the day and night,
The seas obey, at your resistless sway
Now they contract their waters, and expose
The dreary desart of old ocean's reign.
The craggy rocks their horrid sides disclose;
Trembling the sailor views the dreadful scene,
And cautiously the threat'ning ruin shuns.
But where the shallow waters hide the sands,
There ravenous destruction lurks conceal'd,
There the ill-guided vessel falls a prey,
And all her numbers gorge his greedy jaws.
But quick returning see th' impetuous tides
Back to th' abandon'd shores impell the main.
Again the foaming seas extend their waves,
Again the rouling floods embrace the shoars,
And veil the horrours of the empty deep.
Thus the obsequious seas your power confess,
While from the surface healthful vapours rise
Plenteous throughout the atmosphere diffus'd,
Or to supply the mountain's heads with springs,
Or fill the hanging clouds with needful rains,
That friendly streams, and kind refreshing show'rs
May gently lave the sun-burnt thirsty plains,

Or to replenifh all the empty air
With wholfome moifture to increafe the fruits
Of earth, and blefs the labours of mankind.
O Newton, whether flies thy mighty foul,
How fhall the feeble mufe purfue through all
The vaft extent of thy unbounded thought,
That even feeks th' unfeen receffes dark
To penetrate of providence immenfe.
And thou the great difpenfer of the world
Propitious, who with infpiration taught'ft
Our greateft bard to fend thy praifes forth;
Thou, who gav'ft Newton thought; who fmil'dft ferene,
When to its bounds he ftretch'd his fwelling foul;
Who ftill benignant ever bleft his toil,
And deign'd to his enlight'ned mind t' appear
Confefs'd around th' interminated world:
To me O thy divine infufion grant
(O thou in all fo infinitely good)
That I may fing thy everlafting works,
Thy inexhaufted ftore of providence,
In thought effulgent and refounding verfe.
O could I fpread the wond'rous theme around,
Where the wind cools the oriental world,
To the calm breezes of the Zephir's breath,
To where the frozen hyperborean blafts,
To where the boift'rous tempeft-leading fouth
From their deep hollow caves fend forth their ftorms.
Thou ftill indulgent parent of mankind,
Left humid emanations fhould no more
Flow from the ocean, but diffolve away
Through the long feries of revolving time;
And left the vital principle decay,
By which the air fupplies the fprings of life;
Thou haft the fiery vifag'd comets form'd

With vivifying ſpirits all replete,
Which they abundant breathe about the void,
Renewing the prolifick ſoul of things.
No longer now on thee amaz'd we call,
No longer tremble at imagin'd ills,
When comets blaze tremendous from on high,
Or when extending wide their flaming trains
With hideous graſp the skies engirdle round,
And ſpread the terrors of their burning locks.
For theſe through orbits in the length'ning ſpace
Of many tedious rouling years compleat
Around the ſun move regularly on;
And with the planets in harmonious orbs,
And myſtick periods their obeyſance pay
To him majeſtick ruler of the skies
Upon his throne of circled glory fixt.
He or ſome god conſpicuous to the view,
Or elſe the ſubſtitute of nature ſeems,
Guiding the courſes of revolving worlds.
He taught great NEWTON the all-potent laws
Of gravitation, by whoſe ſimple power
The univerſe exiſts. Nor here the ſage
Big with invention ſtill renewing ſtaid.
But O bright angel of the lamp of day,
How ſhall the muſe diſplay his greateſt toil?
Let her plunge deep in Aganippe's waves,
Or in Caſtalia's ever-flowing ſtream,
That re-inſpired ſhe may ſing to thee,
How NEWTON dar'd advent'rous to unbraid
The yellow treſſes of thy ſhining hair.
Or didſt thou gracious leave thy radiant ſphere,
And to his hand thy lucid ſplendours give,
T' unweave the light-diffuſing wreath, and part

The blended glories of thy golden plumes?
He with laborious, and unerring care,
How diff'rent and imbodied colours form
Thy piercing light, with juſt diſtinction found.
He with quick ſight purſu'd thy darting rays,
When penetrating to th' obſcure receſs
Of ſolid matter, there perſpicuous ſaw,
How in the texture of each body lay
The power that ſeparates the diff'rent beams.
Hence over nature's unadorned face
Thy bright diverſifying rays dilate
Their various hues: and hence when vernal rains
Deſcending ſwift have burſt the low'ring clouds,
Thy ſplendors through the diſſipating miſts
In its fair veſture of unnumber'd hues
Array the ſhow'ry bow. At thy approach
The morning riſen from her pearly couch
With roſy bluſhes decks her virgin cheek;
The ev'ning on the frontiſpiece of heav'n
His mantle ſpreads with many colours gay;
The mid-day skies in radiant azure clad,
The ſhining clouds, and ſilver vapours rob'd
In white tranſparent intermixt with gold,
With bright variety of ſplendor cloath
All the illuminated face above.
When hoary-headed winter back retires
To the chill'd pole, there ſolitary ſits
Encompaſs'd round with winds and tempeſts bleak
In caverns of impenetrable ice,
And from behind the diſſipated gloom
Like a new Venus from the parting ſurge
The gay-apparell'd ſpring advances on;
When thou in thy meridian brightneſs ſitt'ſt,
And from thy throne pure emanations flow

Of glory burſting o'er the radiant skies:
Then let the muſe Olympus' top aſcend,
And o'er Theſſalia's plain extend her view,
And count, O Tempe, all thy beauties o'er.
Mountains, whoſe ſummits graſp the pendant clouds,
Between their wood-invelop'd ſlopes embrace
The green-attired vallies. Every flow'r
Here in the pride of bounteous nature clad
Smiles on the boſom of th' enamell'd meads.
Over the ſmiling lawn the ſilver floods
Of fair Peneus gently roul along,
While the reflected colours from the flow'rs,
And verdant borders pierce the lympid waves,
And paint with all their variegated hue
The yellow ſands beneath. Smooth gliding on
The waters haſten to the neighbouring ſea.
Still the pleas'd eye the floating plain purſues;
At length, in Neptune's wide dominion loſt,
Surveys the ſhining billows, that ariſe
Apparell'd each in Phœbus' bright attire:
Or from a far ſome tall majeſtick ſhip,
Or the long hoſtile lines of threat'ning fleets,
Which o'er the bright uneven mirror ſweep,
In dazling gold and waving purple deckt;
Such as of old, when haughty Athens power
Their hideous front, and terrible array
Againſt Pallene's coaſt extended wide,
And with tremendous war and battel ſtern
The trembling walls of Potidæa ſhook.
Creſted with pendants curling with the breeze
The upright maſts high briſtle in the air,
Aloft exalting proud their gilded heads.
The ſilver waves againſt the painted prows
Raiſe their reſplendent boſoms, aud impearl

The fair vermillion with their gliſt'ring drops:
And from on board the iron-ſheathed hoſt
Around the main a gleaming horrour caſts;
Each flaming buckler like the mid-day ſun,
Each plumed helmet like the ſilver moon,
Each moving gauntlet like the light'ning's blaze,
And like a ſtar each brazen pointed ſpear.
But lo the ſacred high-erected fanes,
Fair citadels, and marble-crowned towers,
And ſumptuous palaces of ſtately towns
Magnificent ariſe, upon their heads
Bearing on high a wreath of ſilver light.
But ſee my muſe the high Pierian hill,
Behold its ſhaggy locks and airy top,
Up to the skies th' imperious mountain heaves
The ſhining verdure of the nodding woods.
See where the ſilver Hippocrene flows,
Behold each glitt'ring rivulet, and rill
Through mazes wander down the green deſcent,
And ſparkle through the interwoven trees.
Here reſt a while and humble homage pay,
Here, where the ſacred genius, that inſpir'd
Sublime MÆONIDES and PINDAR's breaſt,
His habitation once was fam'd to hold.
Here thou, O HOMER, offer'dſt up thy vows;
Thee, the kind muſe CALLIOPÆA heard,
And led thee to the empyrean ſeats,
There manifeſted to thy hallow'd eyes
The deeds of gods; thee wiſe MINERVA taught
The wondrous art of knowing human kind;
Harmonious PHOEBUS tun'd thy heav'nly mind,
And ſwell'd to rapture each exalted ſenſe;
Even MARS the dreadful battle-ruling god,
MARS taught thee war, and with his bloody hand

Inſtructed thine, when in thy ſounding lines
We hear the rattling of Bellona's carr,
The yell of diſcord, and the din of arms.
PINDAR, when mounted on his fiery ſteed,
Soars to the ſun, oppoſing eagle like
His eyes undazled to the fierceſt rays.
He firmly ſeated, not like GLAUCUS' ſon,
Strides his ſwift-winged and fire-breathing horſe,
And born aloft ſtrikes with his ringing hoofs
The brazen vault of heav'n, ſuperior there
Looks down upon the ſtars, whoſe radiant light
Illuminates innumerable worlds,
That through eternal orbits roul beneath.
But thou all hail immortalized ſon
Of harmony, all hail thou Thracian bard,
To whom APOLLO gave his tuneful lyre.
O might'ſt thou, ORPHEUS, now again revive,
And NEWTON ſhould inform thy liſt'ning ear
How the ſoft notes, and ſoul-inchanting ſtrains
Of thy own lyre were on the wind convey'd.
He taught the muſe, how ſound progreſſive floats
Upon the waving particles of air,
When harmony in ever-pleaſing ſtrains,
Melodious melting at each lulling fall,
With ſoft alluring penetration ſteals
Through the enraptur'd ear to inmoſt thought,
And folds the ſenſes in its ſilken bands.
So the ſweet muſick, which from ORPHEUS' touch
And fam'd AMPHION's, on the ſounding ſtring
Aroſe harmonious, gliding on the air,
Pierc'd the tough-bark'd and knotty-ribbed woods,
Into their ſaps ſoft inſpiration breath'd
And taught attention to the ſtubborn oak.
Thus when great HENRY, and brave MARLB'ROUGH led

Th'

A POEM on Sir ISAAC NEWTON.

Th' imbattled numbers of BRITANNIA's ſons,
The trump, that ſwells th' expanded cheek of fame,
That adds new vigour to the gen'rous youth,
And rouzes ſluggiſh cowardize it ſelf,
The trumpet with its Mars-inciting voice,
The winds broad breaſt impetuous ſweeping o'er
Fill'd the big note of war. Th' inſpired hoſt
With new-born ardor preſs the trembling GAUL;
Nor greater throngs had reach'd eternal night,
Not if the fields of Agencourt had yawn'd
Expoſing horrible the gulf of fate;
Or roaring Danube ſpread his arms abroad,
And overwhelm'd their legions with his floods.
But let the wand'ring muſe at length return;
Nor yet, angelick genius of the ſun,
In worthy lays her high-attempting ſong
Has blazon'd forth thy venerated name.
Then let her ſweep the loud-reſounding lyre
Again, again o'er each melodious ſtring
Teach harmony to tremble with thy praiſe.
And ſtill thine ear O favourable grant,
And ſhe ſhall tell thee, that whatever charms,
Whatever beauties bloom on nature's face,
Proceed from thy all-influencing light.
That when ariſing with tempeſtuous rage,
The North impetuous rides upon the clouds
Diſperſing round the heav'ns obſtructive gloom,
And with his dreaded prohibition ſtays
The kind effuſion of thy genial beams;
Pale are the rubies on AURORA's lips,
No more the roſes bluſh upon her cheeks,
Black are Peneus' ſtreams and golden ſands
In Tempe's vale dull melancholy ſits,
And every flower reclines its languid head.

By

A Poem on Sir Isaac Newton.

By what high name ſhall I invoke thee, ſay,
Thou life-infuſing deity, on thee
I call, and look propitious from on high,
While now to thee I offer up my prayer.
O had great Newton, as he found the cauſe,
By which ſound rouls thro' th' undulating air,
O had he, baffling times reſiſtleſs power,
Diſcover'd what that ſubtle ſpirit is,
Or whatſoe'er diffuſive elſe is ſpread
Over the wide-extended univerſe,
Which cauſes bodies to reflect the light,
And from their ſtraight direction to divert
The rapid beams, that through their ſurface pierce.
But ſince embrac'd by th' icy arms of age,
And his quick thought by times cold hand congeal'd,
Ev'n Newton left unknown this hidden power;
Thou from the race of human kind ſelect
Some other worthy of an angel's care,
With inſpiration animate his breaſt,
And him inſtruct in theſe thy ſecret laws.
O let not Newton, to whoſe ſpacious view,
Now unobſtructed, all th' extenſive ſcenes
Of the ethereal ruler's works ariſe;
When he beholds this earth he late adorn'd,
Let him not ſee philoſophy in tears,
Like a fond mother ſolitary ſit,
Lamenting him her dear, and only child.
But as the wiſe Pythagoras, and he,
Whoſe birth with pride the fam'd Abdera boaſts,
With expectation having long ſurvey'd
This ſpot their antient ſeat, with joy beheld
Divine philoſophy at length appear
In all her charms majeſtically fair,
Conducted by immortal Newton's hand:

So

So may he see another sage arise,
That shall maintain her empire: then no more
Imperious ignorance with haughty sway
Shall stalk rapacious o'er the ravag'd globe:
Then thou, O NEWTON, shalt protect these lines,
The humble tribute of the grateful muse;
Ne'er shall the sacrilegious hand despoil
Her laurel'd temples, whom his name preserves:
And were she equal to the mighty theme,
Futurity should wonder at her song;
Time should receive her with extended arms,
Seat her conspicuous in his rouling carr,
And bear her down to his extreamest bound.

FABLES with wonder tell how Terra's sons
With iron force unloos'd the stubborn nerves
Of hills, and on the cloud-inshrouded top
Of Pelion Ossa pil'd. But if the vast
Gigantick deeds of savage strength demand
Astonishment from men, what then shalt thou,
O what expressive rapture of the soul,
When thou before us, NEWTON, dost display
The labours of thy great excelling mind;
When thou unveilest all the wondrous scene,
The vast idea of th' eternal king,
Not dreadful bearing in his angry arm
The thunder hanging o'er our trembling heads;
But with th' effulgency of love replete,
And clad with power, which form'd th' extensive heavens.
O happy he, whose enterprizing hand
Unbars the golden and relucid gates
Of th' empyrean dome, where thou enthron'd
Philosophy art seated. Thou sustain'd
By the firm hand of everlasting truth

Despiseſt all the injuries of time:
Thou never know'ſt decay when all around,
Antiquity obſcures her head. Behold
Th' Egyptian towers, the Babylonian walls,
And Thebes with all her hundred gates of braſs,
Behold them ſcatter'd like the duſt abroad.
Whatever now is flouriſhing and proud,
Whatever ſhall, muſt know devouring age.
Euphrates' ſtream, and ſeven-mouthed Nile,
And Danube, thou that from Germania's ſoil
To the black Euxine's far remoted ſhore,
O'er the wide bounds of mighty nations ſweep'ſt
In thunder loud thy rapid floods along.
Ev'n you ſhall feel inexorable time;
To you the fatal day ſhall come; no more
Your torrents then ſhall ſhake the trembling ground,
No longer then to inundations ſwol'n
Th' imperious waves the fertile paſtures drench,
But ſhrunk within a narrow channel glide;
Or through the year's reiterated courſe
When time himſelf grows old, your wond'rous ſtreams
Loſt ev'n to memory ſhall lie unknown
Beneath obſcurity, and Chaos whelm'd.
But ſtill thou ſun illuminateſt all
The azure regions round, thou guideſt ſtill
The orbits of the planetary ſpheres;
The moon ſtill wanders o'er her changing courſe,
And ſtill, O Newton, ſhall thy name ſurvive:
As long as nature's hand directs the world,
When ev'ry dark obſtruction ſhall retire,
And ev'ry ſecret yield its hidden ſtore,
Which thee dim-ſighted age forbad to ſee
Age that alone could ſtay thy riſing ſoul.
And could mankind among the fixed ſtars,

A Poem on Sir Isaac Newton.

E'en to th' extremeſt bounds of knowledge reach,
To thoſe unknown innumerable ſuns,
Whoſe light but glimmers from thoſe diſtant worlds,
Ev'n to thoſe utmoſt boundaries, thoſe bars
That ſhut the entrance of th' illumin'd ſpace
Where angels only tread the vaſt unknown,
Thou ever ſhould'ſt be ſeen immortal there:
In each new ſphere, each new-appearing ſun,
In fartheſt regions at the very verge
Of the wide univerſe ſhould'ſt thou be ſeen.
And lo, th' all-potent goddeſs Nature takes
With her own hand thy great, thy juſt reward
Of immortality; aloft in air
See ſhe diſplays, and with eternal graſp
Uprears the trophies of great Newton's fame.

R. Glover.

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ERRATA.

PAGE 25. line 4. read *In these Precepts*. p. 40. l. 24. for *I* read *K*. p. 53. l. penult. f. *E*. r. *F*. p. 82. l. ult. f. 40. r. 42. p. 83. l. ult. f. 43. r. 45. p. 91. l. 3. f. 48. r. 50. ibid. l. 25. for 49. r. 51. p. 92. l. 18. f. *AGFE*. r. *HGFC*. p. 96. l. 23. dele the comma after $\frac{1}{3}$. p. 140. l. 12. dele *and*. p. 144. l. 15. f. *threefold*. r. *twofold*. p. 162. l. 25. f. $\frac{1}{3}$. r. $\frac{1}{8}$. p. 193. l. 2. r. *always*. p. 199. l. penult. and p. 200. l. 3. 5. f. *F*. r. *C*. p. 201. l. 8. f. *ascends*. r. *must ascend*. ibid. l. 10. f. *it descends*. r. *descend* p. 208. l. 14. f. *WTO*. r. *NTO*. In *fig.* 110. draw a line from *I* through *T*, till it meets the circle *ADCB*, where place *W*. p. 216. l. penult. f. *action*. r. *motion*. p. 221. l. 23. f. *AF*. r. *AH*. p. 232. l. 23. after *invention* put a full point. p. 253. l. penult. dele the comma after *remarkable*. p. 255. l. ult. f. *DE*. r. *BE*. p. 278. l. 17. f. $\xi\tau$. r. $\xi\pi$. p. 299. l. 19. r. *the*. p. 361. l. 12. f. *I*. r. *t*. p. 369. l 2, 3. r. *Pseudo-topaz*. p. 378. l. 12. f. *that*. r. *than*. p. 379. l. 15. f. *converge*. r. *diverge*. p. 384. l. 7. f. *optic-glass*. r. *optic-nerve*. p. 391. l. 18. r. *as* 50 *to* 78. p. 392. l. 18. after *telescope* add *be about* 100 *feet long and the*. in *fig.* 161. f. δ put ϵ. p. 399. l. 8. r. Aν, Aκ, &c. p. 400. l. 19. r. Aπ, Aϱ, Aσ, Aτ, Aϕ. p. 401. l. 14. r. *fig.* 163. The pages 374, 375, 376 are erroneously numbered 375, 376, 377; and the pages 382, 383 are numbered 381, 382.

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INTRODUCTION.

THE manner, in which Sir ISAAC NEWTON has publiſhed his philoſophical diſcoveries, occaſions them to lie very much concealed from all, who have not made the mathematics particularly their ſtudy. He once, indeed, intended to deliver, in a more familiar way, that part of his inventions, which relates to the ſyſtem of the world; but upon farther conſideration he altered his deſign. For as the nature of thoſe diſcoveries made it impoſſible to prove them upon any other than geometrical principles; he apprehended, that thoſe, who ſhould not fully perceive the force of his arguments, would hardly be prevailed on to exchange their former ſentiments for new opinions, ſo very different from

what were commonly received [a]. He therefore chose rather to explain himself only to mathematical readers; and declined the attempting to instruct such in any of his principles, who, by not comprehending his method of reasoning, could not, at the first appearance of his discoveries, have been persuaded of their truth. But now, since Sir ISAAC NEWTON's doctrine has been fully established by the unanimous approbation of all, who are qualified to understand the same; it is without doubt to be wished, that the whole of his improvements in philosophy might be universally known. For this purpose therefore I drew up the following papers, to give a general notion of our great philosopher's inventions to such, as are not prepared to read his own works, and yet might desire to be informed of the progress, he has made in natural knowledge; not doubting but there were many, besides those, whose turn of mind had led them into a course of mathematical studies, that would take great pleasure in tasting of this delightful fountain of science.

2. IT is a just remark, which has been made upon the human mind, that nothing is more suitable to it, than the contemplation of truth; and that all men are moved with a strong desire after knowledge; esteeming it honourable to excel therein; and holding it, on the contrary, disgraceful to mistake, err, or be in any way deceived. And this sentiment is by nothing more fully illustrated, than by the inclination of men to gain an acquaintance with the operations of nature; which disposition to enquire after the causes of things is

[a] Philosoph. Nat. princ. math. L. iii. introduct.

so

ſo general, that all men of letters, I believe, find themſelves influenced by it. Nor is it difficult to aſſign a reaſon for this, if we conſider only, that our deſire after knowledge is an effect of that taſte for the ſublime and the beautiful in things, which chiefly conſtitutes the difference between the human life, and the life of brutes. Theſe inferior animals partake with us of the pleaſures, that immediately flow from the bodily ſenſes and appetites; but our minds are furniſhed with a ſuperior ſenſe, by which we are capable of receiving various degrees of delight, where the creatures below us perceive no difference. Hence ariſes that purſuit of grace and elegance in our thoughts and actions, and in all things belonging to us, which principally creates imployment for the active mind of man. The thoughts of the human mind are too extenſive to be confined only to the providing and enjoying of what is neceſſary for the ſupport of our being. It is this taſte, which has given riſe to poetry, oratory, and every branch of literature and ſcience. From hence we feel great pleaſure in conceiving ſtrongly, and in apprehending clearly, even where the paſſions are not concerned. Perſpicuous reaſoning appears not only beautiful; but, when ſet forth in its full ſtrength and dignity, it partakes of the ſublime, and not only pleaſes, but warms and elevates the ſoul. This is the ſource of our ſtrong deſire of knowledge; and the ſame taſte for the ſublime and the beautiful directs us to chuſe particularly the productions of nature for the ſubject of our contemplation: our creator having ſo adapted our minds to the condition, wherein he has placed us, that all his viſible

works, before we inquire into their make, ſtrike us with the moſt lively ideas of beauty and magnificence.

3. But if there be ſo ſtrong a paſſion in contemplative minds for natural philoſophy; all ſuch muſt certainly receive a particular pleaſure in being informed of Sir Isaac Newton's diſcoveries, who alone has been able to make any great advancements in the true courſe leading to natural knowledge: whereas this important ſubject had before been uſually attempted with that negligence, as cannot be reflected on without ſurprize. Excepting a very few, who, by purſuing a more rational method, had gained a little true knowledge in ſome particular parts of nature; the writers in this ſcience had generally treated of it after ſuch a manner, as if they thought, that no degree of certainty was ever to be hoped for. The cuſtom was to frame conjectures; and if upon comparing them with things, there appeared ſome kind of agreement, though very imperfect, it was held ſufficient. Yet at the ſame time nothing leſs was undertaken than intire ſyſtems, and fathoming at once the greateſt depths of nature; as if the ſecret cauſes of natural effects, contrived and framed by infinite wiſdom, could be ſearched out by the ſlighteſt endeavours of our weak underſtandings. Whereas the only method, that can afford us any proſpect of ſucceſs in this difficult work, is to make our enquiries with the utmoſt caution, and by very ſlow degrees. And after our moſt diligent labour, the greateſt part of nature will, no doubt, for ever remain beyond our reach.

4. This

4. THIS neglect of the proper means to enlarge our knowledge, joined with the presumption to attempt, what was quite out of the power of our limited faculties, the Lord BACON judiciously observes to be the great obstruction to the progress of science [a]. Indeed that excellent person was the first, who expresly writ against this way of philosophizing; and he has laid open at large the absurdity of it in his admirable treatise, intitled NOVUM ORGANON SCIENTIARUM; and has there likewise described the true method, which ought to be followed.

5. THERE are, saith he, but two methods, that can be taken in the pursuit of natural knowledge. One is to make a hasty transition from our first and slight observations on things to general axioms, and then to proceed upon those axioms, as certain and uncontestable principles, without farther examination. The other method; (which he observes to be the only true one, but to his time unattempted;) is to proceed cautiously, to advance step by step, reserving the most general principles for the last result of our inquiries [b]. Concerning the first of these two methods; where objections, which happen to appear against any such axioms taken up in haste, are evaded by some frivolous distinction, when the axiom it self ought rather to be corrected [c]; he affirms, that the united endeavours of all ages cannot make it succesful; because this original error in the first digestion of the mind (as he expresses himself) cannot afterwards be remedied [d]: whereby he would signify to us, that if we set out in a

[a] Nov. Org Scient. L. i. Aphorism. 9.
[b] Nov Org. L. 1. Aph. 19
[c] Ibid. Aph. 25.
[d] Aph. 30 Errores radicales & in prima digestione mentis ab excellentia functionum & remediorum sequentium non curantur.

wrong

wrong way; no diligence or art, we can uſe, while we follow ſo erroneous a courſe, will ever bring us to our deſigned end. And doubtleſs it cannot prove otherwiſe; for in this ſpacious field of nature, if once we forſake the true path, we ſhall immediately loſe our ſelves, and muſt for ever wander with uncertainty.

6. THE impoſſibility of ſucceeding in ſo faulty a method of philoſophizing his Lordſhip endeavours to prove from the many falſe notions and prejudices, to which the mind of man is expoſed [a]. And ſince this judicious writer apprehends, that men are ſo exceeding liable to fall into theſe wrong tracts of thinking, as to incur great danger of being miſled by them, even while they enter on the true courſe in purſuit of nature [b]; I truſt, I ſhall be excuſed, if, by inſiſting a little particularly upon this argument, I endeavour to remove whatever prejudice of this kind, might poſſibly entangle the mind of any of my readers.

7. HIS Lordſhip has reduced theſe prejudices and falſe modes of conception under four diſtinct heads [c].

8. THE firſt head contains ſuch, as we are ſubject to from the very condition of humanity, through the weakneſs both of our ſenſes, and of the faculties of the mind [d]; ſeeing, as this author well obſerves, the ſubtilty of nature far exceeds the greateſt ſubtilty of our ſenſes or acuteſt reaſonings [e]. One

[a] Aph. 38.
[b] Ibid.
[c] Aph. 39.
[d] Aph 41.
[e] Aph. 10, 24.

of the falſe modes of conception, which he mentions under this head, is the forming to our ſelves a fanciful ſimplicity and regularity in natural things. This he illuſtrates by the following inſtances; the conceiving the planets to move in perfect circles; the adding an orb of fire to the other three elements, and the ſuppoſing each of theſe to exceed the other in rarity, juſt in a decuple proportion [a]. And of the ſame nature is the aſſertion of DES CARTES, without any proof, that all things are made up of three kinds of matter only [b]. As alſo this opinion of another philoſopher; that light, in paſſing through different mediums, was refracted, ſo as to proceed by that way, through which it would move more ſpeedily, than through any other [c]. The ſecond erroneous turn of mind, taken notice of by his Lordſhip under this head, is, that all men are in ſome degree prone to a fondneſs for any notions, which they have once imbibed; whereby they often wreſt things to reconcile them to thoſe notions, and neglect the conſideration of whatever will not be brought to an agreement with them; juſt as thoſe do, who are addicted to judicial aſtrology, to the obſervation of dreams, and to ſuch-like ſuperſtitions; who carefully preſerve the memory of every incident, which ſerves to confirm their prejudices, and let ſlip out of their minds all inſtances, that make againſt them [d]. There is alſo a farther impediment to true knowledge, mentioned under the ſame head by this noble writer, which is; that whereas, through the weakneſs and imperfection of our ſenſes, many things are concealed

[a] Aph 45.
[b] Des Cartes Princ Phil. Part. 3. §. 52.
[c] Fermat, in Oper. pag 156, &c.
[d] Nov. Org. Aph. 46.

from

from us, which have the greateſt effect in producing natural appearances; our minds are ordinarily moſt affected by that, which makes the ſtrongeſt impreſſion on our organs of ſenſe; whereby we are apt to judge of the real importance of things in nature by a wrong meaſure [a]. So, becauſe the figuration and the motion of bodies ſtrike our ſenſes more immediately than moſt of their other properties, DES CARTES and his followers will not allow any other explication of natural appearances, than from the figure and motion of the parts of matter. By which example we ſee how juſtly his Lordſhip obſerves this cauſe of error to be the greateſt of any [b]; ſince it has given riſe to a fundamental principle in a ſyſtem of philoſophy, that not long ago obtained almoſt an univerſal reputation.

9. THESE are the chief branches of thoſe obſtructions to knowledge, which this author has reduced under his firſt head of falſe conceptions. The ſecond head contains the errors, to which particular perſons are more eſpecially obnoxious [c]. One of theſe is the conſequence of a preceding obſervation: that as we are expoſed to be captivated by any opinions, which have once taken poſſeſſion of our minds; ſo in particular, natural knowledge has been much corrupted by the ſtrong attachment of men to ſome one part of ſcience, of which they reputed themſelves the inventers, or about which they have ſpent much of their time; and hence have been apt to conceive it to be of greater uſe in the ſtudy of na-

[a] Aph. 50.
[b] Ibid.
[c] Aph. 53.

tural

tural philoſophy than it was: like ARISTOTLE, who reduced his phyſics to logical diſputations; and the chymiſts, who thought, that nature could be laid open only by the force of their fires [a]. Some again are wholly carried away by an exceſſive veneration for antiquity; others, by too great fondneſs for the moderns; few having their minds ſo well balanced, as neither to depreciate the merit of the ancients, nor yet to deſpiſe the real improvements of later times [b]. To this is added by his Lordſhip a difference in the genius of men, that ſome are moſt fitted to obſerve the ſimilitude, there is in things, while others are more qualified to diſcern the particulars, wherein they diſagree; both which diſpoſitions of mind are uſeful: but to the prejudice of philoſophy men are apt to run into exceſs in each; while one ſort of genius dwells too much upon the groſs and ſum of things, and the other upon trifling minuteneſſes and ſhadowy diſtinctions [c].

10. UNDER the third head of prejudices and falſe notions this writer conſiders ſuch, as follow from the lax and indefinite uſe of words in ordinary diſcourſe; which occaſions great ambiguities and uncertainties in philoſophical debates (as another eminent philoſopher has ſince ſhewn more at large [d];) inſomuch that this our author thinks a ſtrict defining of terms to be ſcarce an infallible remedy againſt this inconvenience [e]. And perhaps he has no ſmall reaſon on his ſide: for the common inaccurate ſenſe of words, notwithſtanding the limitations given them by definitions, will offer it ſelf ſo conſtantly to

[a] Aph. 54.
[b] Aph. 56.
[c] Aph. 55.

[d] Locke, On human underſtanding, B. iii.
[e] Nov. Org. Aph. 59.

C the

the mind, as to require great caution and circumſpection for us not to be deceived thereby. Of this we have a very eminent inſtance in the great diſputes, that have been raiſed about the uſe of the word attraction in philoſophy; of which we ſhall be obliged hereafter to make particular mention [a]. Words thus to be guarded againſt are of two kinds. Some are names of things, that are only imaginary [b]; ſuch words are wholly to be rejected. But there are other terms, that allude to what is real, though their ſignification is confuſed [c]. And theſe latter muſt of neceſſity be continued in uſe; but their ſenſe cleared up, and freed, as much as poſſible, from obſcurity.

11. THE laſt general head of theſe errors comprehends ſuch, as follow from the various ſects of falſe philoſophies; which this author divides into three ſorts, the ſophiſtical, empirical, and ſuperſtitious [d]. By the firſt of theſe he means a philoſophy built upon ſpeculations only without experiments [e]; by the ſecond, where experiments are blindly adhered to, without proper reaſoning upon them [f]; and by the third, wrong opinions of nature fixed in mens minds either through falſe religions, or from miſunderſtanding the declarations of the true [g].

12. THESE are the four principal canals, by which this judicious author thinks, that philoſophical errors have flowed in upon us. And he rightly obſerves, that the faulty method of

[a] In the concluſion.
[b] Nov. Org. L. i. Aph. 59.
[c] Ibid Aph. 60.
[d] Ibid. Aph. 62.
[e] Aph. 63.
[f] Aph. 64.
[g] Aph. 65.

proceeding

proceeding in philosophy, against which he writes [a], is so far from assisting us towards overcoming these prejudices; that he apprehends it rather suited to rivet them more firmly to the mind [b]. How great reason then has his Lordship to call this way of philosophizing the parent of error, and the bane of all knowledge [c]? For, indeed, what else but mistakes can so bold and presumptuous a treatment of nature produce? have we the wisdom necessary to frame a world, that we should think so easily, and with so slight a search to enter into the most secret springs of nature, and discover the original causes of things? what chimeras, what monsters has not this preposterous method brought forth? what schemes, or what hypothesis's of the subtilest wits has not a stricter enquiry into nature not only overthrown, but manifested to be ridiculous and absurd? Every new improvement, which we make in this science, lets us see more and more the weakness of our guesses. Dr. HARVEY, by that one discovery of the circulation of the blood, has dissipated all the speculations and reasonings of many ages upon the animal oeconomy. ASELLIUS, by detecting the lacteal veins, shewed how little ground all physicians and philosophers had in conjecturing, that the nutritive part of the aliment was absorbed by the mouths of the veins spread upon the bowels: and then PECQUET, by finding out the thoracic duct, as evidently proved the vanity of the opinion, which was persisted in after the lacteal vessels were known, that the alimental juice was conveyed immediately to the liver, and there converted into blood.

[a] See above, § 4, 5.
[b] Nov. Org. L. i. Aph. 69
[c] Ibid.

13. As these things set forth the great absurdity of proceeding in philosophy on conjectures, by informing us how far the operations of nature are above our low conceptions; so on the other hand, such instances of success from a more judicious method shew us, that our bountiful maker has not left us wholly without means of delighting our selves in the contemplation of his wisdom. That by a just way of inquiry into nature, we could not fail of arriving at discoveries very remote from our apprehensions; the Lord BACON himself argues from the experience of mankind. If, says he, the force of guns should be described to any one ignorant of them, by their effects only; he might reasonably suppose, that those engines of destruction were only a more artificial composition, than he knew, of wheels and other mechanical powers: but it could never enter his thoughts, that their immense force should be owing to a peculiar substance, which would enkindle into so violent an explosion, as we experience in gunpowder: since he would no where see the least example of any such operation; except perhaps in earthquakes and thunder, which he would doubtless look upon as exalted powers of nature, greatly surpassing any art of man to imitate. In the same manner, if a stranger to the original of silk were shewn a garment made of it, he would be very far from imagining so strong a substance to be spun out of the bowels of a small worm; but must certainly believe it either a vegetable substance, like flax or cotton; or the natural covering of some animal, as wool is of sheep. Or had we been told, before the invention of the magnetic needle among us, that another people was in possession of a certain

contrivance

contrivance, by which they were inabled to diſcover the poſition of the heavens, with vaſtly more eaſe, than we could do; what could have been imagined more, than that they were provided with ſome fitter aſtronomical inſtrument for this purpoſe than we? That any ſtone ſhould have ſo amazing a property, as we find in the magnet, muſt have been the remoteſt from our thoughts [a].

14. BUT what ſurprizing advancements in the knowledge of nature may be made by purſuing the true courſe in philoſophical inquiries; when thoſe ſearches are conducted by a genius equal to ſo divine a work, will be beſt underſtood by conſidering Sir ISAAC NEWTON's diſcoveries. That my reader may apprehend as juſt a notion of theſe, as can be conveyed to him, by the brief account, which I intend to lay before him; I have ſet apart this introduction for explaining, in the fulleſt manner I am able, the principles, whereon Sir ISAAC NEWTON proceeds. For without a clear conception of theſe, it is impoſſible to form any true idea of the ſingular excellence of the inventions of this great philoſopher.

15. THE principles then of this philoſophy are; upon no conſideration to indulge conjectures concerning the powers and laws of nature, but to make it our endeavour with all diligence to ſearch out the real and true laws, by which the conſtitution of things is regulated. The philoſopher's firſt care muſt be to diſtinguiſh, what he ſees to be within his power, from what

[a] Ibid. Aph. 109

is beyond his reach; to aſſume no greater degree of knowledge, than what he finds himſelf poſſeſſed of; but to advance by ſlow and cautious ſteps; to ſearch gradually into natural cauſes; to ſecure to himſelf the knowledge of the moſt immediate cauſe of each appearance, before he extends his views farther to cauſes more remote. This is the method, in which philoſophy ought to be cultivated; which does not pretend to ſo great things, as the more airy ſpeculations; but will perform abundantly more: we ſhall not perhaps ſeem to the unskilful to know ſo much, but our real knowledge will be greater. And certainly it is no objection againſt this method, that ſome others promiſe, what is nearer to the extent of our wiſhes: ſince this, if it will not teach us all we could deſire to be informed of, will however give us ſome true light into nature; which no other can do. Nor has the philoſopher any reaſon to think his labour loſt, when he finds himſelf ſtopt at the cauſe firſt diſcovered by him, or at any other more remote cauſe, ſhort of the original: for if he has but ſufficiently proved any one cauſe, he has entered ſo far into the real conſtitution of things, has laid a ſafe foundation for others to work upon, and has facilitated their endeavours in the ſearch after yet more diſtant cauſes; and beſides, in the mean time he may apply the knowledge of theſe intermediate cauſes to many uſeful purpoſes. Indeed the being able to make practical deductions from natural cauſes, conſtitutes the great diſtinction between the true philoſophy and the falſe. Cauſes aſſumed upon conjecture, muſt be ſo looſe and undefined, that nothing particular can be collected from them. But thoſe cauſes, which are brought to light by a ſtrict examination

of

of things, will be more diſtinct. Hence it appears to have been no unuſeful diſcovery, that the aſcent of water in pumps is owing to the preſſure of the air by its weight or ſpring; though the cauſes, which make the air gravitate, and render it elaſtic, be unknown: for notwithſtanding we are ignorant of the original, whence theſe powers of the air are derived; yet we may receive much advantage from the bare knowledge of theſe powers. If we are but certain of the degree of force, wherewith they act, we ſhall know the extent of what is to be expected from them; we ſhall know the greateſt height, to which it is poſſible by pumps to raiſe water; and ſhall thereby be prevented from making any uſeleſs efforts towards improving theſe inſtruments beyond the limits preſcribed to them by nature; whereas without ſo much knowledge as this, we might probably have waſted in attempts of this kind much time and labour. How long did philoſophers buſy themſelves to no purpoſe in endeavouring to perfect teleſcopes, by forming the glaſſes into ſome new figure; till Sir ISAAC NEWTON demonſtrated, that the effects of teleſcopes were limited from another cauſe, than was ſuppoſed; which no alteration in the figure of the glaſſes could remedy? What method Sir ISAAC NEWTON himſelf has found for the improvement of teleſcopes ſhall be explained hereafter [a]. But at preſent I ſhall proceed to illuſtrate, by ſome farther inſtances, this diſtinguiſhing character of the true philoſophy, which we have now under conſideration. It was no trifling diſcovery, that the contraction of the muſcles of animals puts their limbs in motion, though the original cauſe of that contraction

[a] Book III. Chap iv.

remains

remains a ſecret, and perhaps may always do ſo; for the knowledge of thus much only has given riſe to many ſpeculations upon the force and artificial diſpoſition of the muſcles, and has opened no narrow proſpect into the animal fabrick. The finding out, that the nerves are great agents in this action, leads us yet nearer to the original cauſe, and yields us a wider view of the ſubject. And each of theſe ſteps affords us aſſiſtance towards reſtoring this animal motion, when impaired in our ſelves, by pointing out the ſeats of the injuries, to which it is obnoxious. To neglect all this, becauſe we can hitherto advance no farther, is plainly ridiculous. It is confeſſed by all, that GALILEO greatly improved philoſophy, by ſhewing, as we ſhall relate hereafter, that the power in bodies, which we call gravity, occaſions them to move downwards with a velocity equably accelerated [a]; and that when any body is thrown forwards, the ſame power obliges it to deſcribe in its motion that line, which is called by geometers a parabola [b]: yet we are ignorant of the cauſe, which makes bodies gravitate. But although we are unacquainted with the ſpring, whence this power in nature is derived, nevertheleſs we can eſtimate its effects. When a body falls perpendicularly, it is known, how long time it takes in deſcending from any height whatever: and if it be thrown forwards, we know the real path, which it deſcribes; we can determine in what direction, and with what degree of ſwiftneſs it muſt be projected, in order to its ſtriking againſt any object deſired; and we can alſo aſcertain the very force, wherewith it will ſtrike

[a] Book I. Chap. 2. § 14. [b] Ibid § 85. &c.

Sir ISAAC NEWTON has farther taught, that this power of gravitation extends up to the moon, and causes that planet to gravitate as much towards the earth, as any of the bodies, which are familiar to us, would, if placed at the same distance [a]: he has proved likewise, that all the planets gravitate towards the sun, and towards one another; and that their respective motions follow from this gravitation. All this he has demonstrated upon indisputable geometrical principles, which cannot be rendered precarious for want of knowing what it is, which causes these bodies thus mutually to gravitate: any more than we can doubt of the propensity in all the bodies about us, to descend towards the earth; or can call in question the forementioned propositions of GALILEO, which are built upon that principle. And as GALILEO has shewn more fully, than was known before, what effects were produced in the motion of bodies by their gravitation towards the earth; so Sir ISAAC NEWTON, by this his invention, has much advanced our knowledge in the celestial motions. By discovering that the moon gravitates towards the sun, as well as towards the earth; he has laid open those intricacies in the moon's motion, which no astronomer, from observations only, could ever find out [b]: and one kind of heavenly bodies, the comets, have their motion now clearly ascertained; whereof we had before no true knowledge at all [c].

16. DOUBTLESS it might be expected, that such surprizing success should have silenced, at once, every cavil. But we

[a] See Book II. Ch. 3 § 3, 4. of this treatise.
[b] See Book II. Ch. 3. of this treatise.
[c] See Chap 4

D have

have ſeen the contrary. For becauſe this philoſophy profeſſes modeſtly to keep within the extent of our faculties, and is ready to confeſs its imperfections, rather than to make any fruitleſs attempts to conceal them, by ſeeking to cover the defects in our knowledge with the vain oſtentation of raſh and groundleſs conjectures; hence has been taken an occaſion to inſinuate that we are led to miraculous cauſes, and the occult qualities of the ſchools.

17. But the firſt of theſe accuſations is very extraordinary. If by calling theſe cauſes miraculous nothing more is meant than only, that they often appear to us wonderful and ſurprizing, it is not eaſy to ſee what difficulty can be raiſed from thence; for the works of nature diſcover every where ſuch proofs of the unbounded power, and the conſummate wiſdom of their author, that the more they are known, the more they will excite our admiration: and it is too manifeſt to be inſiſted on, that the common ſenſe of the word miraculous can have no place here, when it implies what is above the ordinary courſe of things. The other imputation, that theſe cauſes are occult upon the account of our not perceiving what produces them, contains in it great ambiguity. That ſomething relating to them lies hid, the followers of this philoſophy are ready to acknowledge, nay deſire it ſhould be carefully remarked, as pointing out proper ſubjects for future inquiry. But this is very different from the proceeding of the ſchoolmen in the cauſes called by them occult. For as their occult qualities were underſtood to operate in a manner occult, and not apprehended by us; ſo they were obtruded

truded upon us for ſuch original and eſſential properties in bodies, as made it vain to ſeek any farther cauſe; and a greater power was attributed to them, than any natural appearances authorized. For inſtance, the riſe of water in pumps was aſcribed to a certain abhorrence of a vacuum, which they thought fit to aſſign to nature. And this was ſo far a true obſervation, that the water does move, contrary to its uſual courſe, into the ſpace, which otherwiſe would be left void of any ſenſible matter; and, that the procuring ſuch a vacuity was the apparent cauſe of the water's aſcent. But while we were not in the leaſt informed how this power, called an abhorrence of a vacuum, produced the viſible effects; inſtead of making any advancement in the knowledge of nature, we only gave an artificial name to one of her operations: and when the ſpeculation was puſhed ſo beyond what any appearances required, as to have it concluded, that this abhorrence of a vacuum was a power inherent in all matter, and ſo unlimited as to render it impoſſible for a vacuum to exiſt at all; it then became a much greater abſurdity, in being made the foundation of a moſt ridiculous manner of reaſoning; as at length evidently appeared, when it came to be diſcovered, that this riſe of the water followed only from the preſſure of the air, and extended it ſelf no farther, than the power of that cauſe. The ſcholaſtic ſtile in diſcourſing of theſe occult qualities, as if they were eſſential differences in the very ſubſtances, of which bodies conſiſted, was certainly very abſurd; by reaſon it tended to diſcourage all farther inquiry. But no ſuch ill conſequences can follow from the conſidering of any natural cauſes, which confeſſedly are not traced up to

their firſt original. How ſhall we ever come to the knowledge of the ſeveral original cauſes of things, otherwiſe than by ſtoring up all intermediate cauſes which we can diſcover? Are all the original and eſſential properties of matter ſo very obvious, that none of them can eſcape our firſt view? This is not probable. It is much more likely, that, if ſome of the eſſential properties are diſcovered by our firſt obſervations, a ſtricter examination ſhould bring more to light.

18. But in order to clear up this point concerning the eſſential properties of matter, let us conſider the ſubject a little diſtinctly. We are to conceive, that the matter, out of which the univerſe of things is formed, is furniſhed with certain qualities and powers, whereby it is rendered fit to anſwer the purpoſes, for which it was created. But every property, of which any particle of this matter is in it ſelf poſſeſſed, and which is not barely the conſequence of the union of this particle with other portions of matter, we may call an eſſential property: whereas all other qualities or attributes belonging to bodies, which depend on their particular frame and compoſition, are not eſſential to the matter, whereof ſuch bodies are made; becauſe the matter of theſe bodies will be deprived of thoſe qualities, only by the diſſolution of the body, without working any change in the original conſtitution of one ſingle particle of this maſs of matter. Extenſion we apprehend to be one of theſe eſſential properties, and impenetrability another. Theſe two belong univerſally to all matter; and are the principal ingredients in the idea, which this word matter uſually excites in the mind. Yet as the idea, marked

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by this name, is not purely the creature of our own underſtandings, but is taken for the repreſentation of a certain ſubſtance without us; if we ſhould diſcover, that every part of the ſubſtance, in which we find theſe two properties, ſhould likewiſe be endowed univerſally with any other eſſential qualities; all theſe, from the time they come to our notice, muſt be united under our general idea of matter. How many ſuch properties there are actually in all matter we know not; thoſe, of which we are at preſent apprized, have been found out only by our obſervations on things; how many more a farther ſearch may bring to light, no one can ſay; nor are we certain, that we are provided with ſufficient methods of perception to diſcern them all. Therefore, ſince we have no other way of making diſcoveries in nature, but by gradual inquiries into the properties of bodies; our firſt ſtep muſt be to admit without diſtinction all the properties, which we obſerve; and afterwards we muſt endeavour, as far as we are able, to diſtinguiſh between the qualities, wherewith the very ſubſtances themſelves are indued, and thoſe appearances, which reſult from the ſtructure only of compound bodies. Some of the properties, which we obſerve in things, are the attributes of particular bodies only; others univerſally belong to all, that fall under our notice. Whether ſome of the qualities and powers of particular bodies, be derived from different kinds of matter entring their compoſition, cannot, in the preſent imperfect ſtate of our knowledge, abſolutely be decided; though we have not yet any reaſon to conclude, but that all the bodies, with which we converſe, are framed out of the very ſame kind of matter, and that their diſtinct quali-

qualities are occasioned only by their structure; through the variety whereof the general powers of matter are caused to produce different effects. On the other hand, we should not hastily conclude, that whatever is found to appertain to all matter, which falls under our examination, must for that reason only be an essential property thereof, and not be derived from some unseen disposition in the frame of nature. Sir ISAAC NEWTON has found reason to conclude, that gravity is a property universally belonging to all the perceptible bodies in the universe, and to every particle of matter, whereof they are composed. But yet he no where asserts this property to be essential to matter. And he was so far from having any design of establishing it as such, that, on the contrary, he has given some hints worthy of himself at a cause for it [a]; and expresly says, that he proposed those hints to shew, that he had no such intention [b].

19. IT appears from hence, that it is not easy to determine, what properties of bodies are essentially inherent in the matter, out of which they are made, and what depend upon their frame and composition. But certainly whatever properties are found to belong either to any particular systems of matter, or universally to all, must be considered in philosophy; because philosophy will be otherwise imperfect. Whether those properties can be deduced from some other appertaining to matter, either among those, which are already known, or among such as can be discovered by us, is afterwards to be sought for the farther improvement of our knowledge. But this

[a] At the end of his Optics. in Qu. 21.

[b] See the same treatise, in Advertisement 2.

inquiry

inquiry cannot properly have place in the deliberation about admitting any property of matter or bodies into philoſophy; for that purpoſe it is only to be conſidered, whether the exiſtence of ſuch a property has been juſtly proved or not. Therefore to decide what cauſes of things are rightly received into natural philoſophy, requires only a diſtinct and clear conception of what kind of reaſoning is to be allowed of as convincing, when we argue upon the works of nature.

20. THE proofs in natural philoſophy cannot be ſo abſolutely concluſive, as in the mathematics. For the ſubjects of that ſcience are purely the ideas of our own minds. They may be repreſented to our ſenſes by material objects, but they are themſelves the arbitrary productions of our own thoughts; ſo that as the mind can have a full and adequate knowledge of its own ideas, the reaſoning in geometry can be rendered perfect. But in natural knowledge the ſubject of our contemplation is without us, and not ſo compleatly to be known: therefore our method of arguing muſt fall a little ſhort of abſolute perfection. It is only here required to ſteer a juſt courſe between the conjectural method of proceeding, againſt which I have ſo largely ſpoke; and demanding ſo rigorous a proof, as will reduce all philoſophy to mere ſcepticiſm, and exclude all proſpect of making any progreſs in the knowledge of nature.

21. THE conceſſions, which are to be allowed in this ſcience, are by Sir ISAAC NEWTON included under a very few ſimple precepts.

22. THE firſt is, that more cauſes are not to be received into philoſophy, than are ſufficient to explain the appearances of nature. That this rule is approved of unanimouſly, is evident from thoſe expreſſions ſo frequent among all philoſophers, that nature does nothing in vain; and that a variety of means, where fewer would ſuffice, is needleſs. And certainly there is the higheſt reaſon for complying with this rule. For ſhould we indulge the liberty of multiplying, without neceſſity, the cauſes of things, it would reduce all philoſophy to mere uncertainty; ſince the only proof, which we can have, of the exiſtence of a cauſe, is the neceſſity of it for producing known effects. Therefore where one cauſe is ſufficient, if there really ſhould in nature be two, which is in the laſt degree improbable, we can have no poſſible means of knowing it, and conſequently ought not to take the liberty of imagining, that there are more than one.

23. THE ſecond precept is the direct conſequence of the firſt, that to like effects are to be aſcribed the ſame cauſes. For inſtance, that reſpiration in men and in brutes is brought about by the ſame means; that bodies deſcend to the earth here in EUROPE, and in AMERICA from the ſame principle; that the light of a culinary fire, and of the ſun have the ſame manner of production; that the reflection of light is effected in the earth, and in the planets by the ſame power; and the like.

24. THE third of theſe precepts has equally evident reaſon for it. It is only, that thoſe qualities, which in the ſame body can neither be leſſened nor increaſed, and which belong

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to all bodies that are in our power to make trial upon, ought to be accounted the univerſal properties of all bodies whatever.

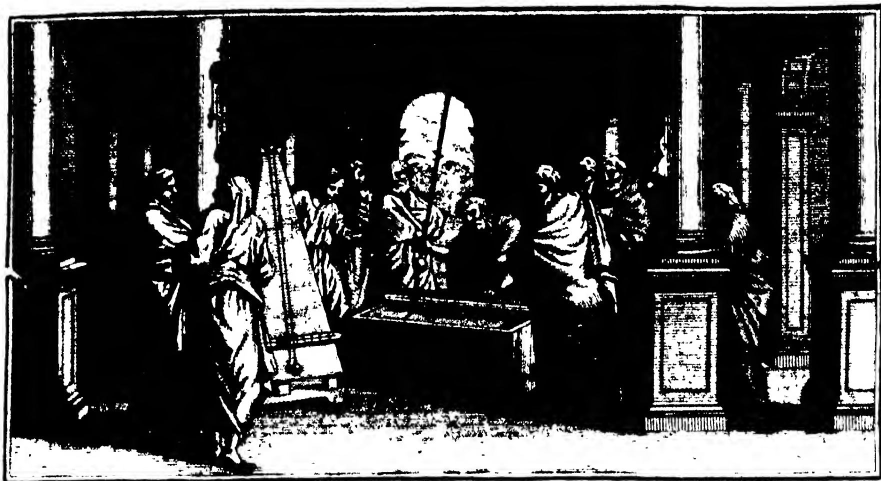
25. In this precept is founded that method of arguing by induction, without which no progreſs could be made in natural philoſophy. For as the qualities of bodies become known to us by experiments only; we have no other way of finding the properties of ſuch bodies, as are out of our reach to experiment upon, but by drawing concluſions from thoſe which fall under our examination. The only caution here required is, that the obſervations and experiments, we argue upon, be numerous enough, and that due regard be paid to all objections, that occur, as the Lord Bacon very judiciouſly directs[a]. And this admonition is ſufficiently complied with, when by virtue of this rule we aſcribe impenetrability and extenſion to all bodies, though we have no ſenſible experiment, that affords a direct proof of any of the celeſtial bodies being impenetrable; nor that the fixed ſtars are ſo much as extended. For the more perfect our inſtruments are, whereby we attempt to find their viſible magnitude, the leſs they appear; inſomuch that all the ſenſible magnitude, which we obſerve in them, ſeems only to be an optical deception by the ſcattering of their light. However, I ſuppoſe no one will imagine they are without any magnitude, though their immenſe diſtance makes it undiſcernable by us. After the ſame manner, if it can be proved, that all

[a] Nov. Org. Lib. i. Ax. 105.

bodies here gravitate towards the earth, in proportion to the quantity of ſolid matter in each; and that the moon gravitates to the earth likewiſe, in proportion to the quantity of matter in it; and that the ſea gravitates towards the moon, and all the planets towards each other; and that the very comets have the ſame gravitating faculty; we ſhall have as great reaſon to conclude by this rule, that all bodies gravitate towards each other. For indeed this rule will more ſtrongly hold in this caſe, than in that of the impenetrability of bodies; becauſe there will more inſtances be had of bodies gravitating, than of their being impenetrable.

25. THIS is that method of induction, whereon all philoſophy is founded; which our author farther inforces by this additional precept, that whatever is collected from this induction, ought to be received, notwithſtanding any conjectural hypotheſis to the contrary, till ſuch times as it ſhall be contradicted or limited by farther obſervations on nature.

BOOK.



BOOK I.

CONCERNING THE MOTION of BODIES IN GENERAL.

CHAP. I.

Of the LAWS of MOTION.

HAVING thus explained Sir ISAAC NEWTON's method of reaſoning in philoſophy, I ſhall now proceed to my intended account of his diſcoveries. Theſe are contained in two treatiſes. In one of them, the MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY, his chief deſign is to ſhew by what laws the heavenly

motions are regulated; in the other, his OPTICS, he difcourfes of the nature of light and colours, and of the action between light and bodies. This fecond treatife is wholly confined to the fubject of light: except fome conjectures propofed at the end concerning other parts of nature, which lie hitherto more concealed. In the other treatife our author was obliged to fmooth the way to his principal intention, by explaining many things of a more general nature: for even fome of the moft fimple properties of matter were fcarce well eftablifhed at that time. We may therefore reduce Sir ISAAC NEWTON's doctrine under three general heads; and I fhall accordingly divide my account into three books. In the firft I fhall fpeak of what he has delivered concerning the motion of bodies, without regard to any particular fyftem of matter; in the fecond I fhall treat of the heavenly motions; and the third fhall be employed upon light.

2. IN the firft part of my defign, we muft begin with an account of the general laws of motion.

3. THESE laws are fome univerfal affections and properties of matter drawn from experience, which are made ufe of as axioms and evident principles in all our arguings upon the motion of bodies. For as it is the cuftom of geometers to affume in their demonftrations fome propofitions, without exhibiting the proof of them; fo in philofophy, all our reafoning muft be built upon fome properties of matter, firft admitted as principles whereon to argue. In geometry thefe axioms are thus affumed, on account of their being fo evident

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as to make any proof in form needless. But in philosophy no properties of bodies can be in this manner received for self-evident; since it has been observed above, that we can conclude nothing concerning matter by any reasonings upon its nature and essence, but that we owe all the knowledge, we have thereof, to experience. Yet when our observations on matter have inform'd us of some of its properties, we may securely reason upon them in our farther inquiries into nature. And these laws of motion, of which I am here to speak, are found so universally to belong to bodies, that there is no motion known, which is not regulated by them. These are by Sir ISAAC NEWTON reduced to three [a].

4. THE first law is, that all bodies have such an indifference to rest, or motion, that if once at rest they remain so, till disturbed by some power acting upon them: but if once put in motion, they persist in it; continuing to move right forwards perpetually, after the power, which gave the motion, is removed; and also preserving the same degree of velocity or quickness, as was first communicated, not stopping or remitting their course, till interrupted or otherwise disturbed by some new power impressed.

5. THE second law of motion is, that the alteration of the state of any body, whether from rest to motion, or from motion to rest, or from one degree of motion to another, is always proportional to the force impressed. A body at rest, when

[a] Princip. philos. pag. 13, 14.

acted

acted upon by any power, yields to that power, moving in the ſame line, in which the power applied is directed ; and moves with a leſs or greater degree of velocity, according to the degree of the power ; ſo that twice the power ſhall communicate a double velocity, and three times the power a threefold velocity. If the body be moving, and the power impreſſed act upon the body in the direction of its motion, the body ſhall receive an addition to its motion, as great as the motion, into which that power would have put it from a ſtate of reſt; but if the power impreſſed upon a moving body act directly oppoſite to its former motion, that power ſhall then take away from the body's motion, as much as in the other caſe it would have added to it. Laſtly, if the power be impreſſed obliquely, there will ariſe an oblique motion differing more or leſs from the former direction, according as the new impreſſion is greater or leſs. For example, if the body A (in fig. 1.) be moving in the direction A B, and when it is at the point A, a power be impreſſed upon it in the direction A C, the body ſhall from henceforth neither move in its firſt direction A B, nor in the direction of the adventitious power, but ſhall take a courſe as A D between them: and if the power laſt impreſſed be juſt equal to that, which firſt gave to the body its motion; the line A D ſhall paſs in the middle between A B and A C, dividing the angle under B A C into two equal parts ; but if the power laſt impreſſed be greater than the firſt, the line AD ſhall incline moſt to AC ; whereas if the laſt impreſſion be leſs than the firſt, the line A D ſhall incline moſt to A B. To be more particular, the ſituation of the

the line A D is always to be determined after this manner. Let A E be the ſpace, which the body would have moved through in the line A B during any certain portion of time; provided that body, when at A, had received no ſecond impulſe. Suppoſe likewiſe, that A F is the part of the line A C, through which the body would have moved during an equal portion of time, if it had been at reſt in A, when it received the impulſe in the direction A C: then if from E be drawn a line parallel to, or equidiſtant from A C, and from F another line parallel to A B, thoſe two lines will meet in the line A D.

6. THE third and laſt of theſe laws of motion is, that when any body acts upon another, the action of that body upon the other is equalled by the contrary reaction of that other body upon the firſt.

7. THESE laws of motion are abundantly confirmed by this, that all the deductions made from them, in relation to the motion of bodies, how complicated ſoever, are found to agree perfectly with obſervation. This ſhall be ſhewn more at large in the next chapter. But before we proceed to ſo diffuſive a proof; I chuſe here to point out thoſe appearances of bodies, whereby the laws of motion are firſt ſuggeſted to us.

8. DAILY obſervation makes it appear to us, that any body, which we once ſee at reſt, never puts it ſelf into freſh motion;

motion; but continues always in the ſame place, till removed by ſome power applied to it.

9. Again, whenever a body is once in motion, it continues in that motion ſome time after the moving power has quitted it, and it is left to it ſelf. Now if the body continue to move but a ſingle moment, after the moving power has left it, there can no reaſon be aſſigned, why it ſhould ever ſtop without ſome external force. For it is plain, that this continuance of the motion is cauſed only by the body's having already moved, the ſole operation of the power upon the body being the putting it in motion; therefore that motion continued will equally be the cauſe of its farther motion, and ſo on without end. The only doubt that can remain, is, whether this motion communicated continues intire, after the power, that cauſed it, ceaſes to act; or whether it does not gradually languiſh and decreaſe. And this ſuſpicion cannot be removed by a tranſient and ſlight obſervation on bodies, but will be fully cleared up by thoſe more accurate proofs of the laws of motion, which are to be conſidered in the next chapter.

10. Lastly, bodies in motion appear to affect a ſtraight courſe without any deviation, unleſs when diſturbed by ſome adventitious power acting upon them. If a body be thrown perpendicularly upwards or downwards, it appears to continue in the ſame ſtraight line during the whole time of its motion. If a body be thrown in any other direction, it is found to deviate from the line, in which it began to move, more and

more

more continually towards the earth, whither it is directed by its weight: but ſince, when the weight of a body does not alter the direction of its motion, it always moves in a ſtraight line, without doubt in this other caſe the body's declining from its firſt courſe is no more, than what is cauſed by its weight alone. As this appears at firſt ſight to be unqueſtionable, ſo we ſhall have a very diſtinct proof thereof in the next chapter, where the oblique motion of bodies will be particularly conſidered.

11. THUS we ſee how the firſt of the laws of motion agrees with what appears to us in moving bodies. But here occurs this farther conſideration, that the real and abſolute motion of any body is not viſible to us: for we are our ſelves alſo in conſtant motion along with the earth whereon we dwell; inſomuch that we perceive bodies to move ſo far only, as their motion is different from our own. When a body appears to us to lie at reſt, in reality it only continues the motion, it has received, without putting forth any power to change that motion. If we throw a body in the courſe or direction, wherein we are carried our ſelves; ſo much motion as we ſeem to have given to the body, ſo much we have truly added to the motion, it had, while it appeared to us to be at reſt. But if we impel a body the contrary way, although the body appears to us to have received by ſuch an impulſe as much motion, as when impelled the other way; yet in this caſe we have taken from the body ſo much real motion, as we ſeem to have given it. Thus the motion, which we ſee in bodies,

is not their real motion, but only relative with reſpect to us; and the forementioned obſervations only ſhew us, that this firſt law of motion has place in this relative or apparent motion. However, though we cannot make any obſervation immediately on the abſolute motion of bodies, yet by reaſoning upon what we obſerve in viſible motion, we can diſcover the properties and effects of real motion.

12. With regard to this firſt law of motion, which is now under conſideration, we may from the foregoing obſervations moſt truly collect, that bodies are diſpoſed to continue in the abſolute motion, which they have once received, without increaſing or diminiſhing their velocity. When a body appears to us to lie at reſt, it really preſerves without change the motion, which it has in common with our ſelves: and when we put it into viſible motion, and we ſee it continue that motion; this proves, that the body retains that degree of its abſolute motion, into which it is put by our acting upon it: if we give it ſuch an apparent motion, which adds to its real motion, it preſerves that addition; and if our acting on the body takes off from its real motion, it continues afterwards to move with no more real motion, than we have left it.

13. Again, we do not obſerve in bodies any diſpoſition or power within themſelves to change the direction of their motion; and if they had any ſuch power, it would eaſily be diſcovered. For ſuppoſe a body by the ſtructure or diſpoſition of its parts, or by any other circumſtance in its make, was indued

dued with a power of moving it ſelf; this ſelf-moving principle, which ſhould be thus inherent in the body, and not depend on any thing external, muſt change the direction wherein it would act, as often as the poſition of the body was changed: ſo that for inſtance, if a body was lying before me in ſuch a poſition, that the direction, wherein this principle diſpoſes the body to move, was pointed directly from me; if I then gradually turned the body about, the direction of this ſelf-moving principle would no longer be pointed directly from me, but would turn about along with the body. Now if any body, which appears to us at reſt, were furniſhed with any ſuch ſelf-moving principle; from the body's appearing without motion we muſt conclude, that this ſelf-moving principle lies directed the ſame way as the earth is carrying the body; and ſuch a body might immediately be put into viſible motion only by turning it about in any degree, that this ſelf-moving principle might receive a different direction.

14. FROM theſe conſiderations it very plainly follows, that if a body were once abſolutely at reſt; not being furniſhed with any principle, whereby it could put it ſelf into motion, it muſt for ever continue in the ſame place, till acted upon by ſomething external: and alſo that when a body is put into motion, it has no power within it ſelf to make any change in the direction of that motion; and conſequently that the body muſt move on ſtraight forward without declining any way whatever. But it has before been ſhewn, that bodies do not appear to have in themſelves any power to

change the velocity of their motion: therefore this firſt law of motion has been illuſtrated and confirmed, as much as can be from the tranſient obſervations, which have here been diſcourſed upon; and in the next chapter all this will be farther eſtabliſhed by more correct obſervations.

15. BUT I ſhall now paſs to the ſecond law of motion; wherein, when it is aſſerted, that the velocity, with which any body is moved by the action of a power upon it, is proportional to that power; the degree of power is ſuppoſed to be meaſured by the greatneſs of the body, which it can move with a given celerity. So that the ſenſe of this law is, that if any body were put into motion with that degree of ſwiftneſs, as to paſs in one hour the length of a thouſand yards; the power, which would give the ſame degree of velocity to a body twice as great, would give this leſſer body twice the velocity, cauſing it to deſcribe in the ſame ſpace of an hour two thouſand yards. But by a body twice as great as another, I do not here mean ſimply of twice the bulk, but one that contains a double quantity of ſolid matter.

16. WHY the power, which can move a body twice as great as another with the ſame degree of velocity, ſhould be called twice as great as the power, which can give the leſſer body the ſame velocity, is evident. For if we ſhould ſuppoſe the greater body to be divided into two equal parts, each equal to the leſſer body, each of theſe halves will require the ſame degree of power to move them with the velocity of the leſſer body, as the leſſer body it ſelf requires; and therefore both

thoſe halves, or the whole greater body, will require the moving power to be doubled.

17. THAT the moving power being in this ſenſe doubled, ſhould juſt double likewiſe the velocity of the ſame body, ſeems near as evident, if we conſider, that the effect of the power applied muſt needs be the ſame, whether that power be applied to the body at once, or in parts. Suppoſe then the double power not applied to the body at once, but half of it firſt, and afterwards the other half; it is not conceivable for what reaſon the half laſt applied ſhould come to have a different effect upon the body, from that which is applied firſt; as it muſt have, if the velocity of the body was not juſt doubled by the application of it. So far as experience can determine, we ſee nothing to favour ſuch a ſuppoſition. We cannot indeed (by reaſon of the conſtant motion of the earth) make trial upon any body perfectly at reſt, whereby to ſee whether a power applied in that caſe would have a different effect, from what it has, when the body is already moving; but we find no alteration in the effect of the ſame power on account of any difference there may be in the motion of the body, when the power is applied. The earth does not always carry bodies with the ſame degree of velocity; yet we find the viſible effects of any power applied to the ſame body to be at all times the very ſame: and a bale of goods, or other moveable body lying in a ſhip is as eaſily removed from place to place, while the ſhip is under ſail, if its motion be ſteady, as when it is fixed at anchor.

18. Now

18. NOW this experience is alone ſufficient to ſhew to us the whole of this law of motion.

19. SINCE we find, that the ſame power will always produce the ſame change in the motion of any body, whether that body were before moving with a ſwifter or ſlower motion; the change wrought in the motion of a body depends only on the power applied to it, without any regard to the body's former motion: and therefore the degree of motion, which the body already poſſeſſes, having no influence on the power applied to diſturb its operation, the effects of the ſame power will not only be the ſame in all degrees of motion of the body; but we have likewiſe no reaſon to doubt, but that a body perfectly at reſt would receive from any power as much motion, as would be equivalent to the effect of the ſame power applied to that body already in motion.

20. AGAIN, ſuppoſe a body being at reſt, any number of equal powers ſhould be ſucceſſively applied to it; puſhing it forward from time to time in the ſame courſe or direction. Upon the application of the firſt power the body would begin to move; when the ſecond power was applied, it appears from what has been ſaid, that the motion of the body would become double; the third power would treble the motion of the body; and ſo on, till after the operation of the laſt power the motion of the body would be as many times the motion, which the firſt power gave it, as there are powers in number. And the effect of this number of powers will be always the ſame,

ſame, without any regard to the ſpace of time taken up in applying them: ſo that greater or leſſer intervals between the application of each of theſe powers will produce no difference at all in their effects. Since therefore the diſtance of time between the action of each power is of no conſequence; without doubt the effect will ſtill be the ſame, though the powers ſhould all be applied at the very ſame inſtant; or although a ſingle power ſhould be applied equal in ſtrength to the collective force of all theſe powers. Hence it plainly follows, that the degree of motion, into which any body will be put out of a ſtate of reſt by any power, will be proportional to that power. A double power will give twice the velocity, a treble power three times the velocity, and ſo on. The foregoing reaſoning will equally take place, though the body were not ſuppoſed to be at reſt, when the powers began to be applied to it; provided the direction, in which the powers were applied, either conſpired with the action of the body, or was directly oppoſite to it. Therefore if any power be applied to a moving body, and act upon the body either in the direction wherewith the body moves, ſo as to accelerate the body; or if it act directly oppoſite to the motion of the body, ſo as to retard it: in both theſe caſes the change of motion will be proportional to the power applied; nay, the augmentation of the motion in one caſe, and the diminution thereof in the other, will be equal to that degree of motion, into which the ſame power would put the body, had it been at reſt, when the power was applied.

21. FARTHER

21. Farther, a power may be ſo applied to a moving body, as to act obliquely to the motion of the body. And the effects of ſuch an oblique motion may be deduced from this obſervation; that as all bodies are continually moving along with the earth, we ſee that the viſible effects of the ſame power are always the ſame, in whatever direction the power acts: and therefore the viſible effects of any power upon a body, which ſeems only to be at reſt, is always to appearance the ſame as the real effect would be upon a body truly at reſt. Now ſuppoſe a body were moving along the line AB (in fig. 2.) and the eye accompanied it with an equal motion in the line CD equidiſtant from AB; ſo that when the body is at A, the eye ſhall be at C, and when the body is advanced to E in the line AB, the eye ſhall be advanced to F in the line CD, the diſtances AE and CF being equal. It is evident, that here the body will appear to the eye to be at reſt; and the line FEG drawn from the eye through the body ſhall ſeem to the eye to be immoveable; though as the body and eye move forward together, this line ſhall really alſo move; ſo that when the body ſhall be advanced to H and the eye to K, the line FEG ſhall be transferred into the ſituation KHL, this line KHL being equidiſtant from FEG. Now if the body when at E were to receive an impulſe in the direction of the line FEG; while the eye is moving on from F to I, and carrying along with it the line FEG, the body will appear to the eye to move along this line FEG: for this is what has juſt now been ſaid; that while bodies are moving along with the earth, and the ſpectator's eye partakes of the ſame motion, the effect of any power upon the body will appear to be what it

it would really have been, had the body been truly at reſt, when the power was applied. From hence it follows, that when the eye is advanced to K, the body will appear ſomewhere in the line KHL. Suppoſe it appear in M; then it is manifeſt, from what has been premiſed at the beginning of this paragraph, that the diſtance HM is equal to what the body would have run upon the line EG, during the time, wherein the eye has paſſed from F to K, provided that the body had been at reſt, when acted upon in E. If it be farther asked, after what manner the body has moved from E to M? I anſwer, through a ſtraight line; for it has been ſhewn above in the explication of the firſt law of motion, that a moving body, from the time it is left to it ſelf, will proceed on in one continued ſtraight line.

22. IF EN be taken equal to HM and NM be drawn; ſince HM is equidiſtant from EN, NM will be equidiſtant from EH. Therefore the effect of any power upon a moving body, when that power acts obliquely to the motion of the body, is to be determined in this manner. Suppoſe the body is moving along the ſtraight line AEB, if when the body is come to E, a power gives it an impulſe in the direction of the line EG, to find what courſe the body will afterwards take we muſt proceed thus. Take in EB any length EH, and in EG take ſuch a length EN, that if the body had been at reſt in E, the power applied to it would have cauſed it to move over EN in the ſame ſpace of time, as it would have employed in paſſing over EH, if the power had not acted at all upon it. Then draw HL equidiſtant from EG, and NM equidiſtant

 from

from EB. After this, if a line be drawn from E to the point M, where these two lines meet, the line EM will be the course into which the body will be put by the action of the power upon it at E.

23. A MATHEMATICAL reader would here expect in some particulars more regular demonstrations; but as I do not at present address my self to such, so I hope, what I have now written will render my meaning evident enough to those, who are unacquainted with that kind of reasoning.

24. Now as we have been shewing, that some actual force is necessary either to put bodies out of a state of rest into motion, or to change the motion, which they have once received; it is proper here to observe, that this quality in bodies, whereby they preserve their present state, with regard to motion or rest, till some active force disturb them, is called the VIS INERTIAE of matter: and by this property, matter, sluggish and unactive of it self, retains all the power impressed upon it, and cannot be made to cease from action, but by the opposition of as great a power, as that which first moved it. By the degree of this VIS INERTIAE, or power of inactivity, as we shall henceforth call it, we primarily judge of the quantity of solid matter in each body; for as this quality is inherent in all the bodies, upon which we can make any trial, we conclude it to be a property essential to all matter; and as we yet know no reason to suppose, that bodies are composed of different kinds of matter, we rather presume, that the matter of all bodies is the same; and that the degree of

this

this power of inactivity is in every body proportional to the quantity of the ſolid matter in it. But although we have no abſolute proof, that all the matter in the univerſe is uniform, and poſſeſſes this power of inactivity in the ſame degree; yet we can with certainty compare together the different degrees of this power of inactivity in different bodies. Particularly this power is proportional to the weight of bodies, as Sir ISAAC NEWTON has demonſtrated [a]. However, notwithſtanding that this power of inactivity in any body can be more certainly known, than the quantity of ſolid matter in it; yet ſince there is no reaſon to ſuſpect that one is not proportional to the other, we ſhall hereafter ſpeak without heſitation of the quantity of matter in bodies, as the meaſure of the degree of their power of inactivity.

25. THIS being eſtabliſhed, we may now compare the effects of the ſame power upon different bodies, as hitherto we have ſhewn the effects of different powers upon the ſame body. And here if we limit the word motion to the peculiar ſenſe given to it in philoſophy, we may comprehend all that is to be ſaid upon this head under one ſhort precept; that the ſame power, to whatever body it is applied, will always produce the ſame degree of motion. But here motion does not ſignify the degree of celerity or velocity with which a body moves, in which ſenſe only we have hitherto uſed it; but it is made uſe of particularly in philoſophy to ſignify the force with which a body moves: as if two bodies A and B be-

[a] Princ. Philoſ. L. II. prop. 24. corol. 7. See alſo B. II. Ch. 5. § 3. of this treatiſe.

ing in motion, twice the force would be required to ſtop A as to ſtop B, the motion of A would be eſteemed double that motion of B. In moving bodies, theſe two things are carefully to be diſtinguiſhed; their velocity or celerity, which is meaſured by the ſpace they paſs through during any determinate portion of time; and the quantity of their motion, or the force, with which they will preſs againſt any reſiſtance. Which force, when different bodies move with the ſame velocity, is proportional to the quantity of ſolid matter in the bodies; but if the bodies are equal, this force is proportional to their reſpective velocities, and in other caſes it is proportional both to the quantity of ſolid matter in the body, and alſo to its velocity. To inſtance in two bodies A and B: if A be twice as great as B, and they have both the ſame velocity, the motion of A ſhall be double the motion of B; and if the bodies be equal, and the velocity of A be twice that of B, the motion of A ſhall likewiſe be double that of B; but if A be twice as large as B, and move twice as ſwift, the motion of A will be four times the motion of B; and laſtly, if A be twice as large as B, and move but half as faſt, the degree of their motion ſhall be the ſame.

26. THIS is the particular ſenſe given to the word motion by philoſophers, and in this ſenſe of the word the ſame power always produces the ſame quantity or degree of motion. If the ſame power act upon two bodies A and B, the velocities, it ſhall give to each of them, ſhall be ſo adjuſted to the reſpective bodies, that the ſame degree of motion ſhall be produced in each. If A be twice as great as B, its velocity ſhall be half

that

that of B; if A has three times as much ſolid matter as B, the velocity of A ſhall be one third of the velocity of B; and generally the velocity given to A ſhall bear the ſame proportion to the velocity given to B, as the quantity of ſolid matter contained in the body B bears to the quantity of ſolid matter contained in A.

27. THE reaſon of all this is evident from what has gone before. If a power were applied to B, which ſhould bear the ſame proportion to the power applied to A, as the body B bears to A, the bodies B and A would both receive the ſame velocity; and the velocity, which B will receive from this power, will bear the ſame proportion to the velocity, which it would receive from the action of the power applied to A, as the former of theſe powers bears to the latter: that is, the velocity, which A receives from the power applied to it, will bear to the velocity, which B would receive from the ſame power, the ſame proportion as the body B bears to A.

28. FROM hence we may now paſs to the third law of motion, where this diſtinction between the velocity of a body and its whole motion is farther neceſſary to be regarded, as ſhall immediately be ſhewn; after having firſt illuſtrated the meaning of this law by a familiar inſtance. If a ſtone or other load be drawn by a horſe; the load re-acts upon the horſe, as much as the horſe acts upon the load; for the harneſs, which is ſtrained between them, preſſes againſt the horſe as much as againſt the load; and the progreſſive motion of the

horſe

horſe forward is hindred as much by the load, as the motion of the load is promoted by the endeavour of the horſe: that is, if the horſe put forth the ſame ſtrength, when looſened from the load, he would move himſelf forwards with greater ſwiftneſs in proportion to the difference between the weight of his own body and the weight of himſelf and load together.

29. This inſtance will afford ſome general notion of the meaning of this law. But to proceed to a more philoſophical explication: if a body in motion ſtrike againſt another at reſt, let the body ſtriking be ever ſo ſmall, yet ſhall it communicate ſome degree of motion to the body it ſtrikes againſt, though the leſs that body be in compariſon of that it impinges upon, and the leſs the velocity is, with which it moves, the ſmaller will be the motion communicated. But whatever degree of motion it gives to the reſting body, the ſame it ſhall loſe it ſelf. This is the neceſſary conſequence of the forementioned power of inactivity in matter. For ſuppoſe the two bodies equal, it is evident from the time they meet, both the bodies are to be moved by the ſingle motion of the firſt; therefore the body in motion by means of its power of inactivity retaining the motion firſt given it, ſtrikes upon the other with the ſame force, wherewith it was acted upon it ſelf: but now both the bodies being to be moved by that force, which before moved one only, the enſuing velocity will be the ſame, as if the power, which was applied to one of the bodies, and put it into motion, had been applied to both; whence it appears, that they will proceed forwards,

that half the velocity, which the body firſt in motion had: that is, the body firſt moved will have loſt half its motion, and the other will have gained exactly as much. This rule is juſt, provided the bodies keep contiguous after meeting; as they would always do, if it were not for a certain cauſe that often intervenes, and which muſt now be explained. Bodies upon ſtriking againſt each other, ſuffer an alteration in their figure, having their parts preſſed inwards by the ſtroke, which for the moſt part recoil again afterwards, the bodies endeavouring to recover their former ſhape. This power, whereby bodies are inabled to regain their firſt figure, is uſually called their elaſticity, and when it acts, it forces the bodies from each other, and cauſes them to ſeparate. Now the effect of this elaſticity in the preſent caſe is ſuch, that if the bodies are perfectly elaſtic, ſo as to recoil with as great a force as they are bent with, that they recover their figure in the ſame ſpace of time, as has been taken up in the alteration made in it by their compreſſion together; then this power will ſeparate the bodies as ſwiftly, as they before approached, and acting upon both equally, upon the body firſt in motion contrary to the direction in which it moves, and upon the other as much in the direction of its motion, it will take from the firſt, and add to the other equal degrees of velocity: ſo that the power being ſtrong enough to ſeparate them with as great a velocity, as they approached with, the firſt will be quite ſtopt, and that which was at reſt, will receive all the motion of the other. If the bodies are elaſtic in a leſs degree, the firſt will not loſe all its motion, nor will the other acquire the motion of the firſt, but fall as much ſhort of it, as the other retains.

For

For this rule is never deviated from, that though the A as of elasticity determines how much more than half its velocity the body first in motion shall lose; yet in every case the loss in the motion of this body shall be transferred to the other, that other body always receiving by the stroke as much motion, as is taken from the first.

30. THIS is the case of a body striking directly against an equal body at rest, and the reasoning here used is fully confirmed by experience. There are many other cases of bodies impinging against one another: but the mention of these shall be reserved to the next chapter, where we intend to be more particular and diffusive in the proof of these laws of motion, than we have been here.

CHAP. II.

Farther proofs of the LAWS of MOTION.

HAVING in the preceding chapter deduced the three laws of motion, delivered by our great philosopher, from the most obvious observations, that suggest them to us; I now intend to give more particular proofs of them, by recounting some of the discoveries which have been made in philosophy before Sir ISAAC NEWTON. For as they were all collected by reasoning upon those laws; so the conformity of these discoveries to experience makes them so many proofs of the truth of the principles, from which they were derived.

2. LET

LET us begin with the ſubject, which concluded the chapter. Although the body in motion be not equal to the body at reſt, on which it ſtrikes; yet the motion after the ſtroke is to be eſtimated in the ſame manner as above. Let A (in fig. 3.) be a body in motion towards another body B lying at reſt. When A is arrived at B, it cannot proceed farther without putting B into motion; and what motion it gives to B, it muſt loſe it ſelf, that the whole degree of motion of A and B together, if neither of the bodies be elaſtic, ſhall be equal, after the meeting of the bodies, to the ſingle motion of A before the ſtroke. Therefore, from what has been ſaid above, it is manifeſt, that as ſoon as the two bodies are met, they will move on together with a velocity, which will bear the ſame proportion to the original velocity of A, as the body A bears to the ſum of both the bodies.

3. IF the bodies are elaſtic, ſo that they ſhall ſeparate after the ſtroke, A muſt loſe a greater part of its motion, and the ſubſequent motion of B will be augmented by this elaſticity, as much as the motion of A is diminiſhed by it. The elaſticity acting equally between both the bodies, it will communicate to each the ſame degree of motion; that is, it will ſeparate the bodies by taking from the body A and adding to the body B different degrees of velocity, ſo proportioned to their reſpective quantities of matter, that the degree of motion, wherewith A ſeparates from B, ſhall be equal to the degree of motion, wherewith B ſeparates from A. It follows therefore, that the velocity taken from A by the elaſticity bears to the velocity, which the ſame elaſticity adds to B, the

H ſame

ſame proportion, as B bears to A: conſequently the vel[illegible] as which the elaſticity takes from A, will bear the ſame propor-tion to the whole velocity, wherewith this elaſticity cauſes the two bodies to ſeparate from each other, as the body B bears to the ſum of the two bodies A and B; and the velocity, which is added to B by the elaſticity, bears to the velocity, where-with the bodies ſeparate, the ſame proportion, as the body A bears to the ſum of the two bodies A and B. Thus is found, how much the elaſticity takes from the velocity of A, and adds to the velocity of B; provided the degree of elaſticity be known, whereby to determine the whole velocity wherewith the bodies ſeparate from each other after the ſtroke [a].

4. After this manner is determined in every caſe the reſult of a body in motion ſtriking againſt another at reſt. The ſame principles will alſo determine the effects, when both bodies are in motion.

5. Let two equal bodies move againſt each other with equal ſwiftneſs. Then the force, with which each of them preſſes forwards, being equal when they ſtrike; each preſſing in its own direction with the ſame energy, neither ſhall ſurmount the other, but both be ſtopt, if they be not elaſtic: for if they be elaſtic, they ſhall from thence recover new motion, and recede from each other, as ſwiftly as they met, if they be perfectly elaſtic; but more ſlowly, if leſs ſo. In the ſame manner, if two bodies of unequal bigneſs ſtrike againſt each other, and their velocities be ſo related, that the velocity

[a] How this degree of elaſticity is to be found by experiment, will be ſhewn below in § 74.

of

that the lesser body shall exceed the velocity of the greater in the same proportion, as the greater body exceeds the lesser (for instance, if one body contains twice the solid matter as the other, and moves but half as fast) two such bodies will entirely suppress each other's motion, and remain from the time of their meeting fixed; if, as before, they are not elastic: but, if they are so in the highest degree, they shall recede again, each with the same velocity, wherewith they met. For this elastic power, as in the preceding case, shall renew their motion, and pressing equally upon both, shall give the same motion to both; that is, shall cause the velocity, which the lesser body receives, to bear the same proportion to the velocity, which the greater receives, as the greater body bears to the lesser: so that the velocities shall bear the same proportion to each other after the stroke, as before. Therefore if the bodies, by being perfectly elastic, have the sum of their velocities after the stroke equal to the sum of their velocities before the stroke, each body after the stroke will receive its first velocity. And the same proportion will hold likewise between the velocities, wherewith they go off, though they are elastic but in a less degree; only then the velocity of each will be less in proportion to the defect of elasticity.

6. IF the velocities, wherewith the bodies meet, are not in the proportion here supposed; but if one of the bodies, as A, has a swifter velocity in comparison to the velocity of the other; then the effect of this excess of velocity in the body A must be joined to the effect now mentioned, after the manner of this following example. Let A be twice as great as B, and

move with the ſame ſwiftneſs as B. Here A moves with that degree of ſwiftneſs, which would anſwer to the forementioned proportion. For A being double to B, if it moved but with half the ſwiftneſs, wherewith B advances, it has been juſt now ſhewn, that the two bodies upon meeting would ſtop, if they were not elaſtic; and if they were elaſtic, that they would each recoil, ſo as to cauſe A to return with half the velocity, wherewith B would return. But it is evident from hence, that B by encountring A will annul half its velocity, if the bodies be not elaſtic; and the future motion of the bodies will be the ſame, as if A had advanced againſt B at reſt with half the velocity here aſſigned to it. If the bodies be elaſtic, the velocity of A and B after the ſtroke may be thus diſcovered. As the two bodies advance againſt each other, the velocity, with which they meet, is made up of the velocities of both bodies added together. After the ſtroke their elaſticity will ſeparate them again. The degree of elaſticity will determine what proportion the velocity, wherewith they ſeparate, muſt bear to that, wherewith they meet. Divide this velocity, with which the bodies ſeparate into two parts, that one of the parts bear to the other the ſame proportion, as the body A bears to B; and aſcribe the leſſer part to the greater body A, and the greater part of the velocity to the leſſer body B. Then take the part aſcribed to A from the common velocity, which A and B would have had after the ſtroke, if they had not been elaſtic; and add the part aſcribed to B to the ſame common velocity. By this means the true velocities of A and B after the ſtroke will be made known.

that 7. IF the bodies are perfectly elastic, the great HUYGENS has laid down this rule for finding their motion after concourse[a]. Any straight line CD (in fig. 4, 5.) being drawn, let it be divided in E, that CE bear the same proportion to ED, as the swiftness of A bore to the swiftness of B before the stroke. Let the same line CD be also divided in F, that CF bear the same proportion to FD, as the body B bears to the body A. Then FG being taken equal to FE, if the point G falls within the line CD, both the bodies shall recoil after the stroke, and the velocity, wherewith the body A shall return, will bear the same proportion to the velocity, wherewith B shall return, as GC bears to GD; but if the point G falls without the line CD, then the bodies after their concourse shall both proceed to move the same way, and the velocity of A shall bear to the velocity of B the same proportion, that GC bears to GD, as before.

8. IF the body B had stood still, and received the impulse of the other body A upon it; the effect has been already explained in the case, when the bodies are not elastic. And when they are elastic, the result of their collision is found by combining the effect of the elasticity with the other effect, in the same manner as in the last case.

9. WHEN the bodies are perfectly elastic, the rule of HUYGENS[b] here is to divide the line CD (fig. 6.) in E as before, and to take EG equal to ED. And by these points

[a] In oper. posthum. de Motu corpor. ex percussion. prop. 9.

[b] In the above cited place

thus

thus found, the motion of each body after the stroke, is determined, as before.

10. In the next place, suppose the bodies A and B were both moving the same way, but A with a swifter motion, so as to overtake B, and strike against it. The effect of the percussion or stroke, when the bodies are not elastic, is discover-ed by finding the common motion, which the two bodies would have after the stroke, if B were at rest, and A were to advance against it with a velocity equal to the excess of the present velocity of A above the velocity of B; and by adding to this common velocity thus found the velocity of B.

11. If the bodies are elastic, the effect of the elasticity is to be united with this other, as in the former cases.

12. When the bodies are perfectly elastic, the rule of Huygens [a] in this case is to prolong CD (fig. 7.) and to take in it thus prolonged CE in the same proportion to ED, as the greater velocity of A bears to the lesser velocity of B; after which FG being taken equal to FE, the velocities of the two bodies after the stroke will be determined, as in the two preceding cases.

13. Thus I have given the sum of what has been written concerning the effects of percussion, when two bodies freely in motion strike directly against each other; and the results here set down, as the consequence of our reasoning

[a] In the place above-cited.

from the laws of motion, anſwer moſt exactly to experience. A particular ſet of experiments has been invented to make trial of theſe effects of percuſſion with the greateſt exactneſs. But I muſt defer theſe experiments, till I have explained the nature of pendulums [a]. I ſhall therefore now proceed to deſcribe ſome of the appearances, which are cauſed in bodies from the influence of the power of gravity united with the general laws of motion; among which the motion of the pendulum will be included.

14. THE moſt ſimple of theſe appearances is, when bodies fall down merely by their weight. In this caſe the body increaſes continually its velocity, during the whole time of its fall, and that in the very ſame proportion as the time increaſes. For the power of gravity acts conſtantly on the body with the ſame degree of ſtrength: and it has been obſerved above in the firſt law of motion, that a body being once in motion will perpetually preſerve that motion without the continuance of any external influence upon it: therefore, after a body has been once put in motion by the force of gravity, the body would continue that motion, though the power of gravity ſhould ceaſe to act any farther upon it; but, if the power of gravity continues ſtill to draw the body down, freſh degrees of motion muſt continually be added to the body; and the power of gravity acting at all times with the ſame ſtrength, equal degrees of motion will conſtantly be added in equal portions of time.

[a] Theſe experiments are deſcribed in § 73.

15. THIS

15. This conclusion is not indeed absolutely true: for we shall find hereafter [a], that the power of gravity is not of the same strength at all distances from the center of the earth. But nothing of this is in the least sensible in any distance, to which we can convey bodies. The weight of bodies is the very same to sense upon the highest towers or mountains, as upon the level ground; so that in all the observations we can make, the forementioned proportion between the velocity of a falling body and the time, in which it has been descending, obtains without any the least perceptible difference.

16. From hence it follows, that the space, through which a body falls, is not proportional to the time of the fall; for since the body increases its velocity, a greater space will be passed over in the same portion of time at the latter part of the fall, than at the beginning. Suppose a body let fall from the point A (in fig. 8.) were to descend from A to B in any portion of time; then if in an equal portion of time it were to proceed from B to C; I say, the space BC is greater than AB; so that the time of the fall from A to C being double the time of the fall from A to B, AC shall be more than double of AB.

17. The geometers have proved, that the spaces, through which bodies fall thus by their weight, are just in a duplicate or two-fold proportion of the times, in which the body has been falling. That is, if we were to take the line DE in the same proportion to AB, as the time, which the body has imployed in falling from A to C, bears to the time of the fall

[a] Book II. Chap 5.

from

from A to B; then AC will be to DE in the ſame proportion. In particular, if the time of the fall through AC be twice the time of the fall through AB; then DE will be twice AB, and AC twice DE; or AC four times AB. But if the time of the fall through AC had been thrice the time of the fall through AB; DE would have been treble of AB, and AC treble of DE; that is, AC would have been equal to nine times AB.

18. IF a body fall obliquely, it will approach the ground by ſlower degrees, than when it falls perpendicularly. Suppoſe two lines AB, AC (in fig. 9.) were drawn, one perpendicular, and the other oblique to the ground DE: then if a body were to deſcend in the ſlanting line AC; becauſe the power of gravity draws the body directly downwards, if the line AC ſupports the body from falling in that manner, it muſt take off part of the effect of the power of gravity; ſo that in the time, which would have been ſufficient for the body to have fallen through the whole perpendicular line AB, the body ſhall not have paſſed in the line AC a length equal to AB; conſequently the line AC being longer than AB, the body ſhall moſt certainly take up more time in paſſing through AC, than it would have done in falling perpendicularly down through AB.

19. THE geometers demonſtrate, that the time, in which the body will deſcend through the oblique ſtraight line AC, bears the ſame proportion to the time of its deſcent through the perpendicular AB, as the line it ſelf AC bears to AB. And in reſpect to the velocity, which the body will have ac-

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quired in the point C, they likewife prove, that the length of the time imployed in the defcent through AC fo compenfates the diminution of the influence of gravity from the obliquity of this line, that though the force of the power of gravity on the body is oppofed by the obliquity of the line AC, yet the time of the body's defcent fhall be fo much prolonged, that the body fhall acquire the very fame velocity in the point C, as it would have got at the point B by falling perpendicularly down.

20. IF a body were to defcend in a crooked line, the time of its defcent cannot be determined in fo fimple a manner; but the fame property, in relation to the velocity, is demonftrated to take place in all cafes: that is, in whatever line the body defcends, the velocity will always be anfwerable to the perpendicular height, from which the body has fell. For inftance, fuppofe the body A (in fig. 10.) were hung by a ftring to the pin B. If this body were let fall, till it came to the point C perpendicularly under B, it will have moved from A to C in the arch of a circle. Then the horizontal line AD being drawn, the velocity of the body in C will be the fame, as if it had fallen from the point D directly down to C.

21. IF a body be thrown perpendicularly upward with any force, the velocity, wherewith the body afcends, fhall continually diminifh, till at length it be wholly taken away; and from that time the body will begin to fall down again, and pafs over a fecond time in its defcent the line, wherein it afcended; falling through this line with an increafing velocity in fuch a manner, that in every point thereof, through which

which it falls, it ſhall have the very ſame velocity, as it had in the ſame place, when it aſcended; and conſequently ſhall come down into the place, whence it firſt aſcended, with the velocity which was at firſt given to it. Thus if a body were thrown perpendicularly up in the line AB (in fig. 11.) with ſuch a force, as that it ſhould ſtop at the point B, and there begin to fall again; when it ſhall have arrived in its deſcent to any point as C in this line, it ſhall there have the ſame velocity, as that wherewith it paſſed by this point C in its aſcent; and at the point A it ſhall have gained as great a velocity, as that wherewith it was firſt thrown upwards. As this is demonſtrated by the geometrical writers; ſo, I think, it will appear evident, by conſidering only, that while the body deſcends, the power of gravity muſt act over again, in an inverted order, all the influence it had on the body in its aſcent; ſo as to give again to the body the ſame degrees of velocity, which it had taken away before.

22. AFTER the ſame manner, if the body were thrown upwards in the oblique ſtraight line CA (in fig. 9.) from the point C, with ſuch a degree of velocity as juſt to reach the point A; it ſhall by its own weight return again through the line AC by the ſame degrees, as it aſcended.

23. AND laſtly, if a body were thrown with any velocity in a line continually incurvated upwards, the like effect will be produced upon its return to the point, whence it was thrown. Suppoſe for inſtance, the body A (in fig. 12.) were hung by a ſtring AB. Then if this body be impelled any

way, it muſt move in the arch of a circle. Let it receive ſuch an impulſe, as ſhall cauſe it to move in the arch A C; and let this impulſe be of ſuch ſtrength, that the body may be carried from A as far as D, before its motion is overcome by its weight: I ſay here, that the body forthwith returning from D, ſhall come again into the point A with the ſame velocity, as that wherewith it began to move.

24. IT will be proper in this place to obſerve concerning the power of gravity, that its force upon any body does not at all depend upon the ſhape of the body; but that it continues conſtantly the ſame without any variation in the ſame body, whatever change be made in the figure of the body: and if the body be divided into any number of pieces, all thoſe pieces ſhall weigh juſt the ſame, as they did, when united together in one body: and if the body be of a uniform contexture, the weight of each piece will be proportional to its bulk. This has given reaſon to conclude, that the power of gravity acts upon bodies in proportion to the quantity of matter in them. Whence it ſhould follow, that all bodies muſt fall from equal heights in the ſame ſpace of time. And as we evidently ſee the contrary in feathers and ſuch like ſubſtances, which fall very ſlowly in compariſon of more ſolid bodies; it is reaſonable to ſuppoſe, that ſome other cauſe concurs to make ſo manifeſt a difference. This cauſe has been found by particular experiments to be the air. The experiments for this purpoſe are made thus. They ſet up a very tall hollow glaſs; within which near the top they lodge a feather and ſome very ponderous body, uſually a piece of gold,

this

this metal being the moſt weighty of any body known to us. This glaſs they empty of the air contained within it, and by moving a wire, which paſſes through the top of the glaſs, they let the feather and the heavy body fall together; and it is always found, that as the two bodies begin to deſcend at the ſame time, ſo they accompany each other in the fall, and come to the bottom at the very ſame inſtant, as near as the eye can judge.—Thus, as far as this experiment can be depended on, it is certain, that the effect of the power of gravity upon each body is proportional to the quantity of ſolid matter, or to the power of inactivity in each body. For in the limited ſenſe, which we have given above to the word motion, it has been ſhewn, that the ſame force gives to all bodies the ſame degree of motion, and different forces communicate different degrees of motion proportional to the reſpective powers [a]. In this caſe, if the power of gravity were to act equally upon the feather, and upon the more ſolid body, the ſolid body would deſcend ſo much ſlower than the feather, as to have no greater degree of motion than the feather: but as both bodies deſcend with equal ſwiftneſs, the degree of motion in the ſolid body is greater than in the feather, bearing the ſame proportion to it, as the quantity of matter in the ſolid body to the quantity of matter in the feather. Therefore the effect of gravity on the ſolid body is greater than on the feather, in proportion to the greater degree of motion communicated; that is, the effect of the power of gravity on the ſolid body bears the ſame proportion to its effect on the feather, as the quanti-

[a] Chap. I. § 25, 26, 27, compared with § 15, &c.

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ty of matter in the ſolid body bears to the quantity of matter in the feather. Thus it is the proper deduction from this experiment, that the power of gravity acts not on the ſurface of bodies only, but penetrates the bodies themſelves moſt intimately, and operates alike on every particle of matter in them. But as the great quickneſs, with which the bodies fall, leaves it ſomething uncertain, whether they do deſcend abſolutely in the ſame time, or only ſo nearly together, that the difference in their ſwift motion is not diſcernable to the eye; this property of the power of gravity, which has here been deduced from this experiment, is farther confirmed by pendulums, whoſe motion is ſuch, that a very minute difference would become ſufficiently ſenſible. This will be farther diſcourſed on in another place [a]; but here I ſhall make uſe of the principle now laid down to explain the nature of what is called the center of gravity in bodies.

25. THE center of gravity is that point, by which if a body be ſuſpended, it ſhall hang at reſt in any ſituation. In a globe of a uniform texture the center of gravity is the ſame with the center of the globe; for as the parts of the globe on every ſide of its center are ſimilarly diſpoſed, and the power of gravity acts alike on every part; it is evident, that the parts of the globe on each ſide of the center are drawn with equal force, and therefore neither ſide can yield to the other; but the globe, if ſupported at its center, muſt of neceſſity hang at reſt. In like manner, if two equal bodies A and B (in

[a] Book II. Chap. 5. § 3.

fig. 13.)

fig. 13.) be hung at the extremities of an inflexible rod C D, which ſhould have no weight; theſe bodies, if the rod be ſupported at its middle E, ſhall equiponderate; and the rod remain without motion. For the bodies being equal and at the ſame diſtance from the point of ſupport E, the power of gravity will act upon each with equal ſtrength, and in all reſpects under the ſame circumſtances; therefore the weight of one cannot overcome the weight of the other. The weight of A can no more ſurmount the weight of B, than the weight of B can ſurmount the weight of A. Again, ſuppoſe a body as A B (in fig. 14.) of a uniform texture in the form of a roller, or as it is more uſually called a cylinder, lying horizontally. If a ſtraight line be drawn between C and D, the centers of the extreme circles of this cylinder; and if this ſtraight line, commonly called the axis of the cylinder, be divided into two equal parts in E: this point E will be the center of gravity of the cylinder. The cylinder being a uniform figure, the parts on each ſide the point E are equal, and ſituated in a perfectly ſimilar manner; therefore this cylinder, if ſupported at the point E, muſt hang at reſt, for the ſame reaſon as the inflexible rod above-mentioned will remain without motion, when ſuſpended at its middle point. And it is evident, that the force applied to the point E, which would uphold the cylinder, muſt be equal to the cylinder's weight. Now ſuppoſe two cylinders of equal thickneſs A B and C D to be joined together at C B, ſo that the two axis's E F, and F G lie in one ſtraight line. Let the axis E F be divided into two equal parts at H, and the axis F G into two equal

equal parts at I. Then becauſe the cylinder AB would be upheld at reſt by a power applied in H equal to the weight of this cylinder, and the cylinder CD would likewiſe be upheld by a power applied in I equal to the weight of this cylinder; the whole cylinder AD will be ſupported by theſe two powers: but the whole cylinder may likewiſe be ſupported by a power applied to K, the middle point of the whole axis EG, provided that power be equal to the weight of the whole cylinder. It is evident therefore, that this power applied in K will produce the ſame effect, as the two other powers applied in H and I. It is farther to be obſerved, that HK is equal to half FG, and KI equal to half EF; for EK being equal to half EG, and EH equal to half EF, the remainder HK muſt be equal to half the remainder FG; ſo likewiſe GK being equal to half GE, and GI equal to half GF, the remainder IK muſt be equal to half the remainder EF. It follows therefore, that HK bears the ſame proportion to KI, as FG bears to EF. Beſides, I believe, my readers will perceive, and it is demonſtrated in form by the geometers, that the whole body of the cylinder CD bears the ſame proportion to the whole body of the cylinder AB, as the axis FG bears to the axis EF [a]. But hence it follows, that in the two powers applied at H and I, the power applied at H bears the ſame proportion to the power applied at I, as KI bears to KH. Now ſuppoſe two ſtrings HL and IM extended upwards, one from the point H and the other from I, and to be laid hold on by two powers, one ſtrong enough to hold up the cylinder AB, and the other of

[a] See Euclid's Elements, Book XII. prop. 13.

ſtrength

ſtrength ſufficient to ſupport the cylinder CD. Here as theſe two powers uphold the whole cylinder, and therefore produce an effect, equal to what would have been produced by a power applied to the point K of ſufficient force to ſuſtain the whole cylinder: it is manifeſt, that if the cylinder be taken away, the axis only being left, and from the point K a ſtring, as KN, be extended, which ſhall be drawn down by a power equivalent to the weight of the cylinder, this power ſhall act againſt the other two powers, as much as the cylinder acted againſt them; and conſequently theſe three powers ſhall be upon a balance, and hold the axis HI fixed between them. But if theſe three powers preſerve a mutual balance, the two powers applied to the ſtrings HL and IM are a balance to each other; the power applied to the ſtring HL bearing the ſame proportion to the power applied to the ſtring IM, as the diſtance IK bears to the diſtance KH. Hence it farther appears, that if an inflexible rod AB (in fig. 15.) be ſuſpended by any point C not in the middle thereof; and if at A the end of the ſhorter arm be hung a weight, and at B the end of the longer arm be alſo hung a weight leſs than the other, and that the greater of theſe weights bears to the leſſer the ſame proportion, as the longer arm of the rod bears to the ſhorter; then theſe two weights will equiponderate: for a power applied at C equal to both theſe weights will ſupport without motion the rod thus charged; ſince here nothing is changed from the preceding caſe but the ſituation of the powers, which are now placed on the contrary ſides of the line, to which they are fixed. Alſo for the

ſame reaſon, if two weights A and B (in fig. 16.) were connected together by an inflexible rod CD, drawn from C the center of gravity of A to D the center of gravity of B; and if the rod CD were to be ſo divided in E, that the part DE bear the ſame proportion to the other part CE, as the weight A bears to the weight B: then this rod being ſupported at E will uphold the weights, and keep them at reſt without motion. This point E, by which the two bodies A and B will be ſupported, is called their common center of gravity. And if a greater number of bodies were joined together, the point, by which they could all be ſupported, is called the common center of gravity of them all. Suppoſe (in fig. 17.) there were three bodies A, B, C, whoſe reſpective centers of gravity were joined by the three lines DE, DF, EF: the line DE being ſo divided in G, that DG bear the ſame proportion to GE, as B bears to A; G is the center of gravity common to the two bodies A and B; that is, a power equal to the weight of both the bodies applied to G would ſupport them, and the point G is preſſed as much by the two weights A and B, as it would be, if they were both hung together at that point. Therefore, if a line be drawn from G to F, and divided in H, ſo that GH bear the ſame proportion to HF, as the weight C bears to both the weights A and B, the point H will be the common center of gravity of all the three weights; for H would be their common center of gravity, if both the weights A and B were hung together at G, and the point G is preſſed as much by them in their preſent ſituation, as it would be in that caſe. In the ſame manner from the common center of theſe three

weights;

weights, you might proceed to find the common center, if a fourth weight were added, and by a gradual progreſs might find the common center of gravity belonging to any number of weights whatever.

26. As all this is the obvious conſequence of the propoſition laid down for aſſigning the common center of gravity of any two weights, by the ſame propoſition the center of gravity of all figures is found. In a triangle, as A B C (in fig. 18.) the center of gravity lies in the line drawn from the middle point of any one of the ſides to the oppoſite angle, as the line B D is drawn from D the middle of the line A C to the oppoſite angle B [a]; ſo that if from the middle of either of the other ſides, as from the point E in the ſide A B, a line be drawn, as E C, to the oppoſite angle; the point F, where this line croſſes the other line B D, will be the center of gravity of the triangle [b]. Likewiſe D F is equal to half F B, and E F equal to half F C [c]. In a hemiſphere, as A B C (fig. 19.) if from D the center of the baſe the line D B be erected perpendicular to that baſe, and this line be ſo divided in E, that D E be equal to three fifths of B E, the point E is the center of gravity of the hemiſphere [d].

27. It will be of uſe to obſerve concerning the center of gravity of bodies; that ſince a power applied to this center alone can ſupport a body againſt the power of gravity, and

[a] Archimed. de æquipond. prop. 11.
[b] Ibid. prop. 12.
[c] Lucas Valerius De centr. gravit. ſolid. L. I. prop. 2.
[d] Idem L. II. prop. 2.

 hold

hold it fixed at reſt; the effect of the power of gravity on a body is the ſame, as if that whole power were to exert itſelf on the center of gravity only. Whence it follows, that, when the power of gravity acts on a body ſuſpended by any point, if the body is ſo ſuſpended, that the center of gravity of the body can deſcend; the power of gravity will give motion to that body, otherwiſe not: or if a number of bodies are ſo connected together, that, when any one is put into motion, the reſt ſhall, by the manner of their being joined, receive ſuch motion, as ſhall keep their common center of gravity at reſt; then the power of gravity ſhall not be able to produce any motion in theſe bodies, but in all other caſes it will. Thus, if the body AB (in fig. 20, 21.) whoſe center of gravity is C, be hung on the point A, and the center C be perpendicularly under A (as in fig. 20.) the weight of the body will hold it ſtill without motion, becauſe the center C cannot deſcend any lower. But if the body be removed into any other ſituation, where the center C is not perpendicularly under A (as in fig. 21.) the body by its weight will be put into motion towards the perpendicular ſituation of its center of gravity. Alſo if two bodies A, B (in fig. 22.) be joined together by the rod CD lying in an horizontal ſituation, and be ſupported at the point E; if this point be the center of gravity common to the two bodies, their weight will not put them into motion; but if this point E is not their common center of gravity, the bodies will move; that part of the rod CD deſcending, in which the common center of gravity is found. So in like manner, if theſe two bodies were connected together by any more complex contrivance; yet

if

if one of the bodies cannot move without ſo moving the other, that their common center of gravity ſhall reſt, the weight of the bodies will not put them in motion, otherwiſe it will.

28. I SHALL proceed in the next place to ſpeak of the mechanical powers. Theſe are certain inſtruments or machines, ~~contrived~~ for the moving great weights with ſmall force; and their effects are all deducible from the obſervation we have juſt been making. They are uſually reckoned in number five; the lever, the wheel and axis, the pulley, the wedge, and the ſcrew; to which ſome add the inclined plane. As theſe inſtruments have been of very ancient uſe, ſo the celebrated ARCHIMEDES ſeems to have been the firſt, who diſcovered the true reaſon of their effects. This, I think, may be collected from what is related of him, that ſome expreſſions, which he uſed to denote the unlimited force of theſe inſtruments, were received as very extraordinary paradoxes: whereas to theſe, who had underſtood the cauſe of their great force, no expreſſions of that kind could have appeared ſurprizing.

29. ALL the effects of theſe powers may be judged of by this one rule, that, when two weights are applied to any of theſe inſtruments, the weights will equiponderate, if, when put into motion, their velocities will be reciprocally proportional to their reſpective weights. And what is ſaid of weights, muſt of neceſſity be equally underſtood of any other forces equi-

equivalent to weights, ſuch as the force of a man's arm, a ſtream of water, or the like.

30. BUT to comprehend the meaning of this rule, the reader muſt know, what is to be underſtood by reciprocal proportion; which I ſhall now endeavour to explain, as diſtinctly as I can; for I ſhall be obliged very frequently to make uſe of this term. When any two things are ſo ~~related~~, that one increaſes in the ſame proportion as the other, they are directly proportional. So if any number of men can perform in a determined ſpace of time a certain quantity of any work, ſuppoſe drain a fiſh-pond, or the like; and twice the number of men can perform twice the quantity of the ſame work, in the ſame time; and three times the number of men can perform as ſoon thrice the work; here the number of men and the quantity of the work are directly proportional. On the other hand, when two things are ſo related, that one decreaſes in the ſame proportion, as the other increaſes, they are ſaid to be reciprocally proportional. Thus if twice the number of men can perform the ſame work in half the time, and three times the number of men can finiſh the ſame in a third part of the time; then the number of men and the time are reciprocally proportional. We ſhewed above [a] how to find the common center of gravity of two bodies, there the diſtances of that common center from the centers of gravity of the two bodies are reciprocally proportional to the reſpective bodies. For CE in fig. 16. being in the ſame pro-

[a] § 25.

portion

portion to ED, as B bears to A; CE is ſo much greater in proportion than ED, as A is leſs in proportion than B.

31. Now this being underſtood, the reaſon of the rule here ſtated will eaſily appear. For if theſe two bodies were put in motion, while the point E reſted, the velocity, wherewith A would move, would bear the ſame proportion to the velocity, wherewith B would move, as EC bears to ED. The velocity therefore of each body, when the common center of gravity reſts, is reciprocally proportional to the body. But we have ſhewn above[a], that if two bodies are ſo connected together, that the putting them in motion will not move their common center of gravity; the weight of thoſe bodies will not produce in them any motion. Therefore in any of theſe mechanical engines, if, when the bodies are put into motion, their velocities are reciprocally proportional to their reſpective weights, whereby the common center of gravity would remain at reſt; the bodies will not receive any motion from their weight, that is, they will equiponderate. But this perhaps will be yet more clearly conceived by the particular deſcription of each mechanical power.

32. THE lever was firſt named above. This is a bar made uſe of to ſuſtain and move great weights. The bar is applied in one part to ſome ſtrong ſupport; as the bar AB (in fig. 23, 24.) is applied at the point C to the ſupport D. In ſome other part of the bar, as E, is applied the weight to be ſuſtained or moved; and in a third place, as F, is applied another weight or equivalent force, which is to ſuſtain or move the

[a] § 27

the weight at E. Now here, if, when the lever ſhould be put in motion, and turned upon the point C, the velocity, wherewith the point F would move, bears the ſame proportion to the velocity, wherewith the point E would move, as the weight at E bears to the weight or force at F; then the lever thus charged will have no propenſity to move either way. If the weight or other force at F be not ſo great as to bear this proportion, the weight at E will not be ſuſtained; but if the force at F be greater than this, the weight at E will be ſurmounted. This is evident from what has been ſaid above[a], when the forces at E and F are placed (as in fig. 23.) on different ſides of the ſupport D. It will appear alſo equally manifeſt in the other caſe, by continuing the bar BC in fig. 24. on the other ſide of the ſupport D, till CG be equal to CF, and by hanging at G a weight equivalent to the power at F; for then, if the power at F were removed, the two weights at G and E would counterpoize each other, as in the former caſe: and it is evident, that the point F will be lifted up by the weight at G with the ſame degree of force, as by the other power applied to F; ſince, if the weight at E were removed, a weight hung at F equal to that at G would balance the lever, the diſtances CG and CF being equal.

33. If the two weights, or other powers, applied to the lever do not counterbalance each other; a third power may be applied in any place propoſed of the lever, which ſhall

[a] Pag. 65, 68.

hold

hold the whole in a juſt counterpoize. Suppoſe (in fig. 25.) the two powers at E and F did not equiponderate, and it were required to apply a third power to the point G, that might be ſufficient to balance the lever. Find what power in F would juſt counterbalance the power in E; then if the difference between this power and that, which is actually applied at F, bear the ſame proportion to the third power to be applied at G, as the diſtance CG bears to CF; the lever will be counterpoized by the help of this third power, if it be ſo applied as to act the ſame way with the power in F, when that power is too ſmall to counterbalance the power in E; but otherwiſe the power in G muſt be ſo applied, as to act againſt the power in F. In like manner, if a lever were charged with three, or any greater number of weights or other powers, which did not counterpoize each other, another power might be applied in any place propoſed, which ſhould bring the whole to a juſt balance. And what is here ſaid concerning a plurality of powers, may be equally applied to all the following caſes.

34. If the lever ſhould conſiſt of two arms making an angle at the point C (as in fig. 26.) yet if the forces are applied perpendicularly to each arm, the ſame proportion will hold between the forces applied, and the diſtances of the center, whereon the lever reſts, from the points to which they are applied. That is, the weight at E will be to the force in F in the ſame proportion, as CF bears to CE.

35. But whenever the forces applied to the lever act obliquely to the arm, to which they are applied (as in fig. 27.)

 then

then the ſtrength of the forces is to be eſtimated by lines let fall from the center of the lever to the directions, wherein the forces act. To balance the levers in fig. 27, the weight or other force at F will bear the ſame proportion to the weight at E, as the diſtance CE bears to CG the perpendicular let fall from the point C upon the line, which denotes the direction wherein the force applied to F acts: for here, if the lever be put into motion, the power applied to F will begin to move in the direction of the line FG; and therefore its firſt motion will be the ſame, as the motion of the point G.

36. When two weights hang upon a lever, and the point, by which the lever is ſupported, is placed in the middle between the two weights, that the arms of the lever are both of equal length; then this lever is particularly called a balance; and equal weights equiponderate as in common ſcales. When the point of ſupport is not equally diſtant from both weights, it conſtitutes that inſtrument for weighing, which is called a ſteelyard. Though both in common ſcales, and the ſteelyard, the point, on which the beam is hung, is not uſually placed juſt in the ſame ſtraight line with the points, that hold the weights, but rather a little above (as in fig. 28.) where the lines drawn from the point C, whereon the beam is ſuſpended, to the points E and F, on which the weights are hung, do not make abſolutely one continued line. If the three points E, C, and F were in one ſtraight line, thoſe weights, which equiponderated, when the beam hung horizontally, would alſo equiponderate in any other ſituation. But we ſee in theſe inſtruments, when they are charged with weights, which

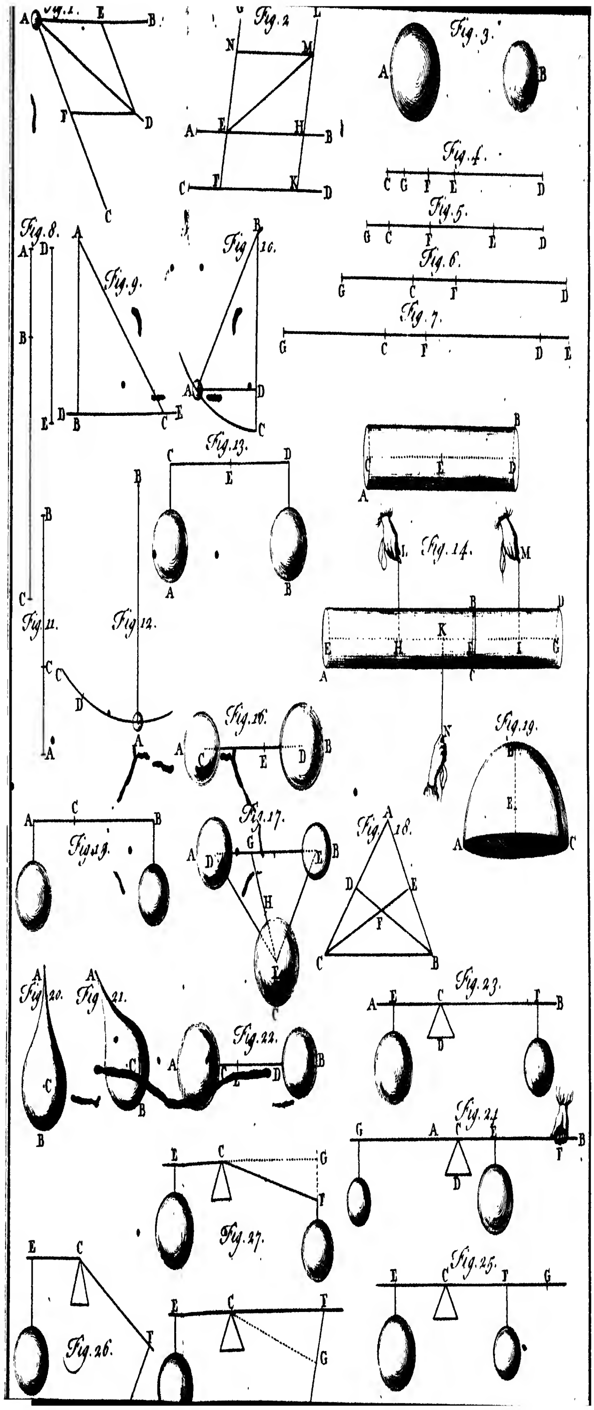
Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig 10.
Fig. 11.
Fig 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig 20.
Fig 21.
Fig. 22.
Fig. 23.
Fig. 24
Fig. 25.
Fig. 26.
Fig. 27.

which equiponderate with the beam hanging horizontally; that, if the beam be inclined either way, the weight moſt elevated ſurmounts the other, and deſcends, cauſing the beam to ſwing, till by degrees it recovers its horizontal poſition. This effect ariſes from the forementioned ſtructure: for by this ſtructure theſe inſtruments are levers compoſed of two arms, which make an angle at the point of ſupport (as in fig. 29, 30.) the firſt of which repreſents the caſe of the common balance, the ſecond the caſe of the ſteelyard. In the firſt, where CE and CF are equal, equal weights hung at E and F will equiponderate, when the points E and F are in an horizontal ſituation. Suppoſe the lines EG and FH to be perpendicular to the horizon, then they will denote the directions, wherein the forces applied to E and F act. Therefore the proportion between the weights at E and F, which ſhall equiponderate, are to be judged of by perpendiculars, as CI, CK, let fall from C upon EG and FH: ſo that the weights being equal, the lines CI, CK, muſt be equal alſo, when the weights equiponderate. But I believe my readers will eaſily ſee, that ſince CE and CF are equal, the lines CI and CK will be equal, when the points E and F are horizontally ſituated.

37. IF this lever be ſet into any other poſition (as in fig. 31.) then the weight, which is raiſed higheſt, will outweigh the other. Here, if the point F be raiſed higher than E, the perpendicular CK will be longer than CI: and therefore the weights would equiponderate, if the weight at F

 were

were leſs than the weight at E. But the weight at F is equal to that at E; therefore is greater, than is neceſſary to counterbalance the weight at E, and conſequently will outweigh it and draw the beam of the lever down.

38. In like manner in the caſe of the ſteelyard (fig. 32.) if the weights at E and F are ſo proportioned, as to equiponderate, when the points E and F are horizontally ſituated; then in any other ſituation of this lever the weight, which is raiſed higheſt, will preponderate. That is, if in the horizontal ſituation of the points E and F the weight at F bears the ſame proportion to the weight at E, as CI bears to CK; then, if the point F be raiſed higher than E (as in fig. 32.) the weight at F ſhall bear a greater proportion to the weight at E, than CI bears to CK.

39. Farther a lever may be hung upon an axis, and then the two arms of the lever need not be continued, but fixed to different parts of this axis; as in fig. 33, where the axis AB is ſupported by its two extremities A and B. To this axis one arm of the lever is fixed at the point C, the other at the point D. Now here, if a weight be hung at E, the extremity of that arm, which is fixed to the axis at the point C; and another weight be hung at F, the extremity of the arm, which is fixed on the axis at D; then theſe weights will equiponderate, when the weight at E bears the ſame proportion to the weight at F, as the arm DF bears to CE.

40. The

40. THIS is the caſe, if both the arms are perpendicular to the axis, and lie (as the geometers expreſs themſelves) in the ſame plane; or, in other words, if the arms are ſo fixed perpendicularly upon the axis, that, when one of them lies horizontally, the other ſhall alſo be horizontal. If either arm ſtand not perpendicular to the axis; then, in determining the proportion between the weights, inſtead of the length of that arm, you muſt uſe the perpendicular let fall upon the axis from the extremity of that arm. If the arms are not ſo fixed as to become horizontal, at the ſame time; the method of aſſigning the proportion between the weights is analogous to that made uſe of above in levers, which make an angle at the point, whereon they are ſupported.

41. FROM this caſe of the lever hung on an axis, it is eaſy to make a tranſition to another mechanical power, the wheel and axis.

42. THIS inſtrument is a wheel fixed on a roller, the roller being ſupported at each extremity ſo as to turn round freely with the wheel, in the manner repreſented in fig. 34, where AB is the wheel, CD the roller, and EF its two ſupports. Now ſuppoſe a weight G hung by a cord wound round the roller, and another weight H hung by a cord wound about the wheel the contrary way: that theſe weights may ſupport each other, the weight H muſt bear the ſame proportion to the weight G, as the thickneſs of the roller bears to the diameter of the wheel.

43. SUPPOSE

43. Suppose the line kl to be drawn through the middle of the roller; and from the place of the roller, where the cord, on which the weight G hangs, begins to leave the roller, as at m, let the line mn be drawn perpendicularly to kl; and from the point, where the cord holding the weight H begins to leave the wheel, as at o, let the line op be drawn perpendicular to kl. This being done, the two lines op and mn reprefent two arms of a lever fixed on the axis kl; confequently the weight H will bear to the weight G the fame proportion, as mn bears to op. But mn bears the fame proportion to op, as the thicknefs of the roller bears to the diameter of the wheel; for mn is half the thicknefs of the roller, and op half the diameter of the wheel.

44. If the wheel be put into motion, and turned once round, that the cord, on which the weight G hangs, be wound once more round the axis; then at the fame time the cord, whereon the weight H hangs, will be wound off from the wheel one circuit. Therefore the velocity of the weight G will bear the fame proportion to the velocity of the weight H, as the circumference of the roller to the circumference of the wheel. But the circumference of the roller bears the fame proportion to the circumference of the wheel, as the thicknefs of the roller bears to the diameter of the wheel, confequently the velocity of the weight G bears to the velocity of the weight H the fame proportion, as the thicknefs of the roller bears to the diameter of the wheel, which is the proportion that the weight H bears to the weight G. Therefore as before in the lever, fo here alfo the general rule laid down

down above is verified, that the weights equiponderate, when their velocities would be reciprocally proportional to their reſpective weights.

45. IN like manner, if on the ſame axis two wheels of different ſizes are fixed (as in fig. 35.) and a weight hung on each; the weights will equiponderate, if the weight hung on the greater wheel bear the ſame proportion to the weight hung on the leſſer, as the diameter of the leſſer wheel bears to the diameter of the greater.

46. IT is uſual to join many wheels together in the ſame frame, which by the means of certain teeth, formed in the circumference of each wheel, ſhall communicate motion to each other. A machine of this nature is repreſented in fig. 36. Here A B C is a winch, upon which is fixed a ſmall wheel D indented with teeth, which move in the like teeth of a larger wheel E F fixed on the axis G H. Let this axis carry another wheel I, which ſhall move in like manner a greater wheel K L fixed on the axis M N. Let this axis carry another ſmall wheel O, which after the ſame manner ſhall turn about a larger wheel P Q fixed on the roller R S, on which a cord ſhall be wound, that holds a weight, as T. Now the proportion required between the weight T and a power applied to the winch at A ſufficient to ſupport the weight, will moſt eaſily be eſtimated, by computing the proportion, which the velocity of the point A would bear to the velocity of the weight. If the winch be turned round, the point A will deſcribe a circle as A V. Suppoſe the wheel E F to have ten times the number of teeth, as the

the wheel D; then the winch muſt turn round ten times to carry the wheel EF once round. If the wheel KL has alſo ten times the number of teeth, as I, the wheel I muſt turn round ten times to carry the wheel KL once round; and conſequently the winch ABC muſt turn round an hundred times to turn the wheel KL once round. Laſtly, if the wheel PQ has ten times the number of teeth, as the wheel O, the winch muſt turn about one thouſand times in order to turn the wheel PQ, or the roller RS once round. Therefore here the point A muſt have gone over the circle AV a thouſand times, in order to lift the weight T through a ſpace equal to the circumference of the roller RS: whence it follows, that the power applied at A will balance the weight T, if it bear the ſame proportion to it, as the circumference of the roller to one thouſand times the circle AV; or the ſame proportion as half the thickneſs of the roller bears to one thouſand times AB.

47. I SHALL now explain the effect of the pulley. Let a weight hang by a pulley, as in fig. 37. Here it is evident, that the power A, by which the weight B is ſupported, muſt be equal to the weight; for the cord CD is equally ſtrained between them; and if the weight B move, the power A muſt move with equal velocity. The pulley E has no other effect, than to permit the power A to act in another direction, than it muſt have done, if it had been directly applied to ſupport the weight without the intervention of any ſuch inſtrument.

48. AGAIN, let a weight be ſupported, as in fig. 38; where the weight A is fixed to the pulley B, and the cord, by which

which the weight is upheld, is annexed by one extremity to a hook C, and at the other end is held by the power D. Here the weight is ſupported by a cord doubled; inſomuch that although the cord were not ſtrong enough to hold the weight ſingle, yet being thus doubled it might ſupport it. If the end of the cord held by the power D were hung on the hook C, as well as the other end; then, when both ends of the cord were tied to the hook, it is evident, that the hook would bear the whole weight; and each end of the ſtring would bear againſt the hook with the force of half the weight only, ſeeing both ends together bear with the force of the whole. Hence it is evident, that, when the power D holds one end of the weight, the force, which it muſt exert to ſupport the weight, muſt be equal to juſt half the weight. And the ſame proportion between the weight and power might be collected from comparing the reſpective velocities, with which they would move; for it is evident, that the power muſt move through a ſpace equal to twice the diſtance of the pulley from the hook, in order to lift the pulley up to the hook.

49. IT is equally eaſy to eſtimate the effect, when many pulleys are combined together, as in fig. 39, 40; in the firſt of which the under ſet of pulleys, and conſequently the weight is held by ſix ſtrings; and in the latter figure by five: therefore in the firſt of theſe figures the power to ſupport the weight, muſt be one ſixth part only of the weight, and in the latter figure the power muſt be one fifth part.

 50. THERE

50. There are two other ways of ſupporting a weight by pulleys, which I ſhall particularly conſider.

51. One of theſe ways is repreſented in fig. 41. Here the weight being connected to the pulley B, a power equal to half the weight A would ſupport the pulley C, if applied immediately to it. Therefore the pulley C is drawn down with a force equal to half the weight A. But if the pulley D were to be immediately ſupported by half the force, with which the pulley C is drawn down, this pulley D will uphold the pulley C; ſo that if the pulley D be upheld with a force equal to one fourth part of the weight A, that force will ſupport the weight. But, for the ſame reaſon as before, if the power in E be equal to half the force neceſſary to uphold the pulley D; this pulley, and conſequently the weight A, will be upheld: therefore, if the power in E be one eighth part of the weight A, it will ſupport the weight.

52. Another way of applying pulleys to a weight is repreſented in fig. 42. To explain the effect of pulleys thus applied, it will be proper to conſider different weights hanging, as in fig. 43. Here, if the power and weights balance each other, the power A is equal to the weight B; the weight C is equal to twice the power A, or the weight B; and for the ſame reaſon the weight D is equal to twice the weight C, or equal to four times the power A. It is evident therefore, that all the three weights B, C, D together are equal to ſeven times the power A. But if theſe three weights were joined in one, they would produce the caſe of fig. 40: ſo that in that figure the weight

weight A, where there are three pulleys, is ſeven times the power B. If there had been but two pulleys, the weight would have been three times the power; and if there had ben four pulleys, the weight would have been fifteen times the power.

53. THE wedge is next to be conſidered. The form of this inſtrument is ſufficiently known. When it is put under any weight (as in fig. 44.) the force, with which the wedge will lift the weight, when drove under it by a blow upon the end AB, will bear the ſame proportion to the force, wherewith the blow would act on the weight, if directly applied to it; as the velocity, which the wedge receives from the blow, bears to the velocity, wherewith the weight is lifted by the wedge.

54. THE ſcrew is the fifth mechanical power. There are two ways of applying this inſtrument. Sometimes it is ſcrewed into a hole, as in fig. 45, where the ſcrew AB is ſcrewed through the plank CD. Sometimes the ſcrew is applied to the teeth of a wheel, as in fig. 46, where the thred of the ſcrew AB turns in the teeth of a wheel CD. In both theſe caſes, if a bar, as AE, be fixed to the end A of the ſcrew; the force, wherewith the end B of the ſcrew in fig. 45 is forced down, and the force, wherewith the teeth of the wheel CD in fig. 44 are held, bears the ſame proportion to the power applied to the end E of the bar; as the velocity, wherewith the end E will move, when the ſcrew is turned, bears to the velocity, wherewith the end B of the ſcrew in fig. 43, or the teeth of the wheel CD in fig. 46, will be moved.

55. THE inclined plane affords alſo a means of raiſing a weight with leſs force, than what is equal to the weight itſelf. Suppoſe it were required to raiſe the globe A (in fig. 47.) from the ground BC up to the point, whoſe perpendicular height from the ground is ED. If this globe be drawn along the ſlant DF, leſs force will be required to raiſe it, than if it were lifted directly up. Here if the force applied to the globe bear the ſame proportion only to its weight, as ED bears to FD, it will be ſufficient to hold up the globe; and therefore any addition to that force will put it in motion, and draw it up; unleſs the globe, by preſſing againſt the plane, whereon it lies, adhere in ſome degree to the plane. This indeed it muſt always do more or leſs, ſince no plane can be made ſo abſolutely ſmooth as to have no inequalities at all; nor yet ſo infinitely hard, as not to yield in the leaſt to the preſſure of the weight. Therefore the globe cannot be laid on ſuch a plane, whereon it will ſlide with perfect freedom, but they muſt in ſome meaſure rub againſt each other; and this friction will make it neceſſary to imploy a certain degree of force more, than what is neceſſary to ſupport the globe, in order to give it any motion. But as all the mechanical powers are ſubject in ſome degree or other to the like impediment from friction; I ſhall here only ſhew what force would be neceſſary to ſuſtain the globe, if it could lie upon the plane without cauſing any friction at all. And I ſay, that if the globe were drawn by the cord GH, lying parallel to the plane DF; and the force, wherewith the cord is pulled, bear the ſame proportion to the weight of the globe, as ED bears to DF; this

this force will ſuſtain the globe. In order to the making proof of this, let the cord GH be continued on, and turned over the pulley I, and let the weight K be hung to it. Now I ſay, if this weight bears the ſame proportion to the globe A, as DE bears to DF, the weight will ſupport the globe. I think it is very manifeſt, that the center of the globe A will lie in one continued line with the cord HG. Let L be the center of the globe, and M the center of gravity of the weight K. In the firſt place let the weight hang ſo, that a line drawn from L to M ſhall lie horizontally; and I ſay, if the globe be moved either up or down the plane DF, the weight will ſo move along with it, that the center of gravity common to both the weights ſhall continue in this line LM, and therefore ſhall in no caſe deſcend. To prove this more fully, I ſhall depart a little from the method of this treatiſe, and make uſe of a mathematical propoſition or two: but they are ſuch, as any perſon, who has read EUCLID's ELEMENTS, will fully comprehend; and are in themſelves ſo evident, that, I believe, my readers, who are wholly ſtrangers to geometrical writings, will make no difficulty of admitting them. This being premiſed, let the globe be moved up, till its center be at G, then will M the center of gravity of the weight K be ſunk to N; ſo that MN ſhall be equal to GL. Draw NG croſſing the line ML in O; then I ſay, that O is the common center of gravity of the two weights in this their new ſituation. Let GP be drawn perpendicular to ML; then GL will bear the ſame proportion to GP, as DF bears to DE; and MN being equal to GL, MN will bear the ſame proportion

to

to GP, as DF bears to DE. But NO bears the ſame proportion to OG, as MN bears to GP; conſequently NO will bear the ſame proportion to OG, as DF bears to DE. In the laſt place, the weight of the globe A bears the ſame proportion to the other weight K, as DF bears to DE; therefore NO bears the ſame proportion to OG, as the weight of the globe A bears to the weight K. Whence it follows, that, when the center of the globe A is in G, and the center of gravity of the weight K is in N, O will be the center of gravity common to both the weights. After the ſame manner, if the globe had been cauſed to deſcend, the common center of gravity would have been found in this line ML. Since therefore no motion of the globe either way will make the common center of gravity deſcend, it is manifeſt, from what has been ſaid above, that the weights A and K counterpoize each other.

56. I SHALL now conſider the caſe of pendulums. A pendulum is made by hanging a weight to a line, ſo that it may ſwing backwards and forwards. This motion the geometers have very carefully conſidered, becauſe it is the moſt commodious inſtrument of any for the exact meaſurement of time.

57. I HAVE obſerved already [a], that if a body hanging perpendicularly by a ſtring, as the body A (in fig. 48.) hangs by the ſtring AB, be put ſo into motion, as to be made to aſcend up the circular arch AC; then as ſoon as it has arrived

[a] § 23.

at the higheſt point, to which the motion, that the body has received, will carry it; it will immediately begin to deſcend, and at A will receive again as great a degree of motion, as it had at firſt. This motion therefore will carry the body up the arch AD, as high as it aſcended before in the arch AC. Conſequently in its return through the arch DA it will acquire again at A its original velocity, and advance a ſecond time up the arch AC as high as at firſt; by this means continuing without end its reciprocal motion. It is true indeed, that in fact every pendulum, which we can put in motion, will gradually leſſen its ſwing, and at length ſtop, unleſs there be ſome power conſtantly applied to it, whereby its motion ſhall be renewed; but this ariſes from the reſiſtance, which the body meets with both from the air, and the ſtring by which it is hung: for as the air will give ſome obſtruction to the progreſs of the body moving through it; ſo alſo the ſtring, whereon the body hangs, will be a farther impediment; for this ſtring muſt either ſlide on the pin, whereon it hangs, or it muſt bend to the motion of the weight; in the firſt there muſt be ſome degree of friction, and in the latter the ſtring will make ſome reſiſtance to its inflection. However, if all reſiſtance could be removed, the motion of a pendulum would be perpetual.

58. BUT to proceed, the firſt property, I ſhall take notice of in this motion, is, that the greater arch the pendulous body moves through, the greater time it takes up: though the length of time does not increaſe in ſo great a proportion as the arch. Thus if CD be a greater arch, and EF a leſſer, where CA is equal to AD, and EA equal to AF; the body,

when

when it ſwings through the greater arch C D, ſhall take up in its ſwing from C to D a longer time than in ſwinging from E to F, when it moves only in that leſſer arch; or the time in which the body let fall from C will deſcend through the arch C A is greater than the time, in which it will deſcend through the arch E A, when let fall from E. But the firſt of theſe times will not hold the ſame proportion to the latter, as the firſt arch C A bears to the other arch E A; which will appear thus. Let C G and E H be two horizontal lines. It has been remarked above [a], that the body in falling through the arch C A will acquire as great a velocity at the point A, as it would have gained by falling directly down through G A; and in falling through the arch E A it will acquire in the point A only that velocity, which it would have got in falling through H A. Therefore, when the body deſcends through the greater arch C A, it ſhall gain a greater velocity, than when it paſſes only through the leſſer; ſo that this greater velocity will in ſome degree compenſate the greater length of the arch.

59. The increaſe of velocity, which the body acquires in falling from a greater height, has ſuch an effect, that, if ſtraight lines be drawn from A to C and E, the body would fall through the longer ſtraight line C A juſt in the ſame time, as through the ſhorter ſtraight line E A. This is demonſtrated by the geometers, who prove, that if any circle, as ABCD (fig. 49.) be placed in a perpendicular ſituation; a body ſhall fall obliquely through every line, as A B drawn from the loweſt point A in the circle to any other point in the circumference

[a] § 20.

ference juſt in the ſame time, as would be imployed by the body in falling perpendicularly down through the diameter C A. But the time in which the body will deſcend through the arch, is different from the time, which it would take up in falling through the line A B.

60. IT has been thought by ſome, that becauſe in very ſmall arches, this correſpondent ſtraight line differs but little from the arch itſelf; therefore the deſcent through this ſtraight line would be performed in ſuch ſmall arches nearly in the ſame time as through the arches themſelves: ſo that if a pendulum were to ſwing in ſmall arches, half the time of a ſingle ſwing would be nearly equal to the time, in which a body would fall perpendicularly through twice the length of the pendulum. That is, the whole time of the ſwing, according to this opinion, will be four fold the time required for the body to fall through half the length of the pendulum; becauſe the time of the body's falling down twice the length of the pendulum is half the time required for the fall through one quarter of this ſpace, that is through half the pendulum's length. However there is here a miſtake; for the whole time of the ſwing, when the pendulum moves through ſmall arches, bears to the time required for a body to fall down through half the length of the pendulum very nearly the ſame proportion, as the circumference of a circle bears to its diameter; that is very nearly the proportion of 355 to 113, or little more than the proportion of 3 to 1. If the pendulum takes ſo great a ſwing, as to paſs over an arch equal to one ſixth part of the whole circumference of the

N circle,

circle, it will ſwing 115 times, while it ought according to this proportion to have ſwung 117 times; ſo that, when it ſwings in ſo large an arch, it loſes ſomething leſs than two ſwings in an hundred. If it ſwing through $\frac{1}{10}$ only of the circle, it ſhall not loſe above one vibration in 160. If it ſwing in $\frac{1}{20}$ of the circle, it ſhall loſe about one vibration in 690. If its ſwing be confined to $\frac{1}{40}$ of the whole circle, it ſhall loſe very little more than one ſwing in 2600. And if it take no greater a ſwing than through $\frac{1}{60}$ of the whole circle, it ſhall not loſe one ſwing in 5800.

61. NOW it follows from hence, that, when pendulums ſwing in ſmall arches, there is very nearly a conſtant proportion obſerved between the time of their ſwing, and the time, in which a body would fall perpendicularly down through half their length. And we have declared above, that the ſpaces, through which bodies fall, are in a two fold proportion of the times, which they take up in falling[a]. Therefore in pendulums of different lengths, ſwinging through ſmall arches, the lengths of the pendulums are in a two fold or duplicate proportion of the times, they take in ſwinging; ſo that a pendulum of four times the length of another ſhall take up twice the time in each ſwing, one of nine times the length will make one ſwing only for three ſwings of the ſhorter, and ſo on.

62. THIS proportion in the ſwings of different pendulums not only holds in ſmall arches; but in large ones alſo,

[a] § 17.

provided

provided they be ſuch, as the geometers call ſimilar; that is, if the arches bear the ſame proportion to the whole circumferences of their reſpective circles. Suppoſe (in fig. 48.) A B, C D to be two pendulums. Let the arch E F be deſcribed by the motion of the pendulum A B, and the arch G H be deſcribed by the pendulum C D; and let the arch E F bear the ſame proportion to the whole circumference, which would be formed by turning the pendulum A B quite round about the point A, as the arch G H bears to the whole circumference, that would be formed by turning the pendulum C D quite round the point C. Then I ſay, the proportion, which the length of the pendulum A B bears to the length of the pendulum C D, will be two fold of the proportion, which the time taken up in the deſcription of the arch E F bears to the time employed in the deſcription of the arch G H.

63. THUS pendulums, which ſwing in very ſmall arches, are nearly an equal meaſure of time. But as they are not ſuch an equal meaſure to geometrical exactneſs; the mathematicians have found out a method of cauſing a pendulum ſo to ſwing, that, if its motion were not obſtructed by any reſiſtance, it would always perform each ſwing in the ſame time, whether it moved through a greater, or a leſſer ſpace. This was firſt diſcovered by the great HUYGENS, and is as follows. Upon the ſtraight line A B (in fig. 49.) let the circle C D E be ſo placed, as to touch the ſtraight line in the point C. Then let this circle roll along upon the ſtraight line A B, as a coach-wheel rolls along upon the ground. It is evident, that, as

ſoon as ever the circle begins to move, the point C in the circle will be lifted off from the ſtraight line A B; and in the motion of the circle will deſcribe a crooked courſe, which is repreſented by the line C F G H. Here the part C H of the ſtraight line included between the two extremities C and H of the line C F G H will be equal to the whole circumference of the circle C D E; and if C H be divided into two equal parts at the point I, and the ſtraight line I K be drawn perpendicular to C H, this line I K will be equal to the diameter of the circle C D E. Now in this line if a body were to be let fall from the point H, and were to be carried by its weight down the line H G K, as far as the point K, which is the loweſt point of the line C F G H; and if from any other point G a body were to be let fall in the ſame manner; this body, which falls from G, will take juſt the ſame time in coming to K, as the body takes up, which falls from H. Therefore if a pendulum can be ſo hung, that the ball ſhall move in the line A G F E, all its ſwings, whether long or ſhort, will be performed in the ſame time; for the time, in which the ball will deſcend to the point K, is always half the time of the whole ſwing. But the ball of a pendulum will be made to ſwing in this line by the following means. Let K I (in fig. 52.) be prolonged upwards to L, till I L is equal to I K. Then let the line L M H equal and like to K H be applied, as in the figure between the points L and H, ſo that the point which in this line L M H anſwers to the point H in the line K H ſhall be applied to the point L, and the point anſwering to the point K ſhall be applied to the point H. Alſo let ſuch another line L N C be applied between L and C in the ſame manner.

manner. This preparation being made; if a pendulum be hung at the point L of ſuch a length, that the ball thereof ſhall reach to K; and if the ſtring ſhall continually bend againſt the lines HML and LNC, as the pendulum ſwings to and fro; by this means the ball ſhall conſtantly keep in the line CKH.

64. Now in this pendulum, as all the ſwings, whether long or ſhort, will be performed in the ſame time; ſo the time of each will exactly bear the ſame proportion to the time required for a body to fall perpendicularly down, through half the length of the pendulum, that is from I to K, as the circumference of a circle bears to its diameter.

65. It may from hence be underſtood in ſome meaſure, why, when pendulums ſwing in circular arches, the times of their ſwings are nearly equal, if the arches are ſmall, though thoſe arches be of very unequal lengths; for if with the ſemidiameter LK the circular arch OKP be deſcribed, this arch in the lower part of it will differ very little from the line CKH.

66. It may not be amiſs here to remark, that a body will fall in this line CKH (fig. 53.) from C to any other point, as Q or R in a ſhorter ſpace of time, than if it moved through the ſtraight line drawn from C to the other point; or through any other line whatever, that can be drawn between theſe two points.

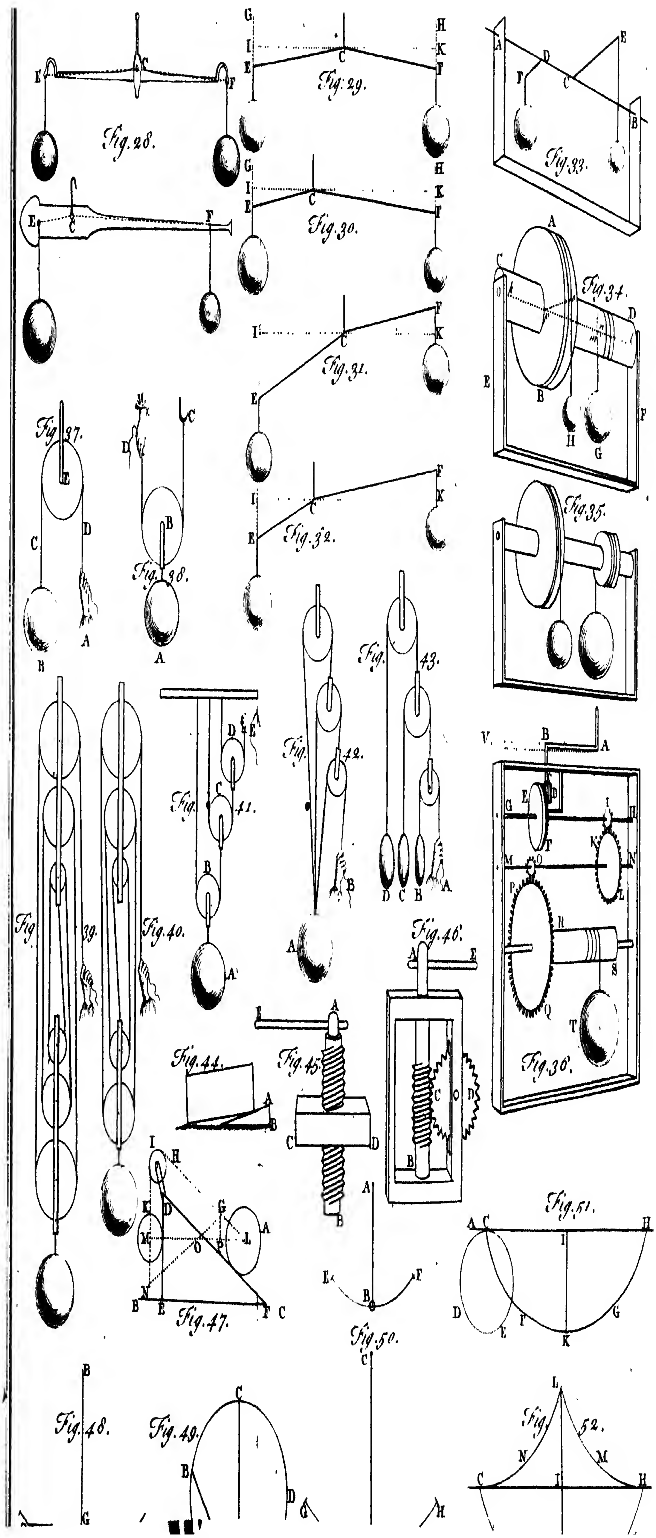
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Fig. 32.
Fig. 33.
Fig. 34.
Fig. 35.
Fig. 36.
Fig. 37.
Fig. 38.
Fig. 39.
Fig. 40.
Fig. 41.
Fig. 42.
Fig. 43.
Fig. 44.
Fig. 45.
Fig. 46.
Fig. 47.
Fig. 48.
Fig. 49.
Fig. 50.
Fig. 51.
Fig. 52.

been obſerved above, that the effect of gravity upon any body is the ſame, as if the whole force were exerted on the body's center of gravity [a]. Since therefore the power of gravity in drawing down the body muſt alſo communicate to it the rolling motion juſt deſcribed; it ſeems evident, that the center of gravity of the body cannot be drawn down as ſwiftly, as when the power of gravity has no other effect to produce on the body, than merely to draw it downward. If therefore the whole matter of the body CB could be crouded into its center of gravity, ſo that being united into one point, this rolling motion here mentioned might give no hindrance to its deſcent; this center would deſcend faſter, than it can now do. And the point, which now deſcends as faſt, as if the whole matter of the body CB were crouded into it, will be farther removed from the point A, than the center of gravity of the body CB.

69. AGAIN, ſuppoſe the pendulum AB (in fig. 55.) to hang obliquely. Here the power of gravity will operate leſs upon the ball of the pendulum, than before: but the line DE being drawn ſo, as to ſtand perpendicular to the rod AC of the pendulum; the force of gravity upon the body CB, now it is in this ſituation, will produce the ſame effect, as if the body were to glide down an inclined plane in the poſition of DE. But here the motion of the body, when the rod is fixed to the point A, will not be equal to the uninterrupted deſcent of the body down this plane; for the body

[a] § 27.

will here alſo receive the ſame kind of rotation in its motion, as before; ſo that the motion of the center of gravity will in like manner be retarded; and the point, which here deſcends with that degree of ſwiftneſs, which the body would have, if not hindered by being fixed to the point A; that is, the point, which deſcends as faſt, as if the whole body were crouded into it, will be as far removed from the point A, as before.

70. THIS point, by which the length of the pendulum is to be eſtimated, is called the center of oſcillation. And the mathematicians have laid down general directions, whereby to find this center in all bodies. If the globe AB (in fig. 56.) be hung by the ſtring CD, whoſe weight need not be regarded, the center of oſcillation is found thus. Let the ſtraight line drawn from C to D be continued through the globe to F. That it will paſs through the center of the globe is evident. Suppoſe E to be this center of the globe; and take the line G of ſuch a length, that it ſhall bear the ſame proportion to ED, as ED bears to EC. Then EH being made equal to $\frac{2}{5}$ of G, the point H ſhall be the center of oſcillation[a]. If the weight of the rod CD is too conſiderable to be neglected, divide CD (fig. 57) in I, that DI be equal to $\frac{1}{3}$, part of CD; and take K in the ſame proportion to CI, as the weight of the globe AB to the weight of the rod CD. Then having found H, the center of oſcillation of the globe, as before, divide IK in L, ſo that IL ſhall bear the ſame pro-

[a] Hugen. Horolog. oſcillat. pag. 141, 142.

portion

portion to LH, as the line CH bears to K; and L ſhall be the center of oſcillation of the whole pendulum.

71. THIS computation is made upon ſuppoſition, that the center of oſcillation of the rod CD, if that were to ſwing alone without any other weight annexed, would be the point I. And this point would be the true center of oſcillation, ſo far as the thickneſs of the rod is not to be regarded. If any one chuſes to take into conſideration the thickneſs of the rod, he muſt place the center of oſcillation thereof ſo much below the point I, that eight times the diſtance of the center from the point I ſhall bear the ſame proportion to the thickneſs of the rod, as the thickneſs of the rod bears to its length CD[a].

72. IT has been obſerved above, that when a pendulum ſwings in an arch of a circle, as here in fig. 58, the pendulum AB ſwings in the circular arch CD; if you draw an horizontal line, as EF, from the place whence the pendulum is let fall, to the line AG, which is perpendicular to the horizon: then the velocity, which the pendulum will acquire in coming to the point G, will be the ſame, as any body would acquire in falling directly down from F to G. Now this is to be underſtood of the circular arch, which is deſcribed by the center of oſcillation of the pendulum. I ſhall here farther obſerve, that if the ſtraight line EG be drawn from the point, whence the pendulum falls, to the loweſt point of the arch; in the ſame or in equal pendulums the velocity, which the

[a] See Hugen. Horolog. Oſcillat. p. 142.

pendulum acquires in G, is proportional to this line: that is, if the pendulum, after it has defcended from E to G, be taken back to H, and let fall from thence, and the line HG be drawn; the velocity, which the pendulum fhall acquire in G by its defcent from H, fhall bear the fame proportion to the velocity, which it acquires in falling from E to G, as the ftraight line HG bears to the ftraight line EG.

73. We may now proceed to thofe experiments upon the percuffion of bodies, which I obferved above might be made with pendulums. This expedient for examining the effects of percuffion was firft propofed by our late great architect Sir Christopher Wren. And it is as follows. Two balls, as A and B (in fig. 59.) either equal or unequal, are hung by two ftrings from two points C and D, fo that, when the balls hang down without motion, they fhall juft touch each other, and the ftrings be parallel. Here if one of thefe balls be removed to any diftance from its perpendicular fituation, and then let fall to defcend and ftrike againft the other; by the laft preceding paragraph it will be known, with what velocity this ball fhall return into its firft perpendicular fituation, and confequently with what force it fhall ftrike againft the other ball; and by the height to which this other ball afcends after the ftroke, the velocity communicated to this ball will be difcovered. For inftance, let the ball A be taken up to E, and from thence be let fall to ftrike againft B, paffing over in its defcent the circular arch EF. By this impulfe let B fly up to G, moving through the circular arch HG. Then EI and GK being drawn horizontally, the

the ball A will ſtrike againſt B with the velocity, which it would acquire in falling directly down from I; and the ball B has received a velocity, wherewith, if it had been thrown directly upward, it would have aſcended up to K. Likewiſe if ſtraight lines be drawn from E to F and from H to G, the velocity of A, wherewith it ſtrikes, will bear the ſame proportion to the velocity, which B has received by the blow, as the ſtraight line EF bears to the ſtraight line HG. In the ſame manner by noting the place to which A aſcends after the ſtroke, its remaining velocity may be compared with that, wherewith it ſtruck againſt B. Thus may be experimented the effects of the body A ſtriking againſt B at reſt. If both the bodies are lifted up, and ſo let fall as to meet and impinge againſt each other juſt upon the coming of both into their perpendicular ſituation; by obſerving the places into which they move after the ſtroke, the effects of their percuſſion in all theſe caſes may be found in the ſame manner as before.

74. Sir ISAAC NEWTON has deſcribed theſe experiments; and has ſhewn how to improve them to a greater exactneſs by making allowance for the reſiſtance, which the air gives to the motion of the balls [a]. But as this reſiſtance is exceeding ſmall, and the manner of allowing for it is delivered by himſelf in very plain terms, I need not enlarge upon it here. I ſhall rather ſpeak to a diſcovery, which he made by theſe experiments upon the elaſticity of bodies. It has been explained above [b], that when two bodies ſtrike, if they be not elaſtic,

[a] Princip. Philoſ. pag. 22.   [b] Chap. 1. § 29.

they remain contiguous after the ſtroke; but that if they are elaſtic, they ſeparate, and that the degree of their elaſticity determines the proportion between the celerity wherewith they ſeparate, and the celerity wherewith they meet. Now our author found, that the degree of elaſticity appeared in the ſame bodies always the ſame, with whatever degree of force they ſtruck; that is, the celerity wherewith they ſeparated, always bore the ſame proportion to the celerity wherewith they met: ſo that the elaſtic power in all the bodies, he made trial upon, exerted it ſelf in one conſtant proportion to the compreſſing force. Our author made trial with balls of wool bound up very compact, and found the celerity with which they receded, to bear about the proportion of 5 to 9 to the celerity wherewith they met; and in ſteel he found nearly the ſame proportion; in cork the elaſticity was ſomething leſs; but in glaſs much greater; for the celerity, wherewith balls of that material ſeparated after percuſſion, he found to bear the proportion of 15 to 16 to the celerity wherewith they met [a].

75. I SHALL finiſh my diſcourſe on pendulums, with this farther obſervation only, that the center of oſcillation is alſo the center of another force. If a body be fixed to any point, and being put in motion turns round it; the body, if uninterrupted by the power of gravity or any other means, will continue perpetually to move about with the ſame equable motion. Now the force, with which ſuch a body

[a] Princip. Philoſ. pag. 25.

moves, is all united in the point, which in relation to the power of gravity is called the center of oſcillation. Let the cylinder ABCD (in fig. 60.) whoſe axis is EF, be fixed to the point E. And ſuppoſing the point E to be that on which the cylinder is ſuſpended, let the center of oſcillation be found in the axis EF, as has been explained above [a]. Let G be that center: then I ſay, that the force, wherewith this cylinder turns round the point E, is ſo united in the point G, that a ſufficient force applied in that point ſhall ſtop the motion of the cylinder, in ſuch a manner, that the cylinder ſhould immediately remain without motion, though it were to be looſened from the point E at the ſame inſtant, that the impediment was applied to G: whereas, if this impediment had been applied to any other point of the axis, the cylinder would turn upon the point, where the impediment was applied. If the impediment had been applied between E and G, the cylinder would ſo turn on the point, where the impediment was applied, that the end BC would continue to move on the ſame way it moved before along with the whole cylinder; but if the impediment were applied to the axis farther off from E than G, the end AD of the cylinder would ſtart out of its preſent place that way in which the cylinder moved. From this property of the center of oſcillation, it is alſo called the center of percuſſion. That excellent mathematician, Dr. BROOK TAYLOR, has farther improved this doctrine concerning the center of percuſſion, by ſhewing, that if through this point G a line, as GHI, be drawn perpendicular to EF, and lying

[a] § 71.

in

in the courſe of the body's motion; a ſufficient power applied to any point of this line will have the ſame effect, as the like power applied to G[a]: ſo that as we before ſhewed the center of percuſſion within the body on its axis; by this means we may find this center on the ſurface of the body alſo, for it will be where this line H I croſſes that ſurface.

76. I shall now proceed to the laſt kind of motion, to be treated on in this place, and ſhew what line the power of gravity will cauſe a body to deſcribe, when it is thrown forwards by any force. This was firſt diſcovered by the great Galileo, and is the principle, upon which engineers ſhould direct the ſhot of great guns. But as in this caſe bodies deſcribe in their motion one of thoſe lines, which in geometry are called conic ſections; it is neceſſary here to premiſe a deſcription of thoſe lines. In which I ſhall be the more particular, becauſe the knowledge of them is not only neceſſary for the preſent purpoſe, but will be alſo required hereafter in ſome of the principal parts of this treatiſe.

77. The firſt lines conſidered by the ancient geometers were the ſtraight line and the circle. Of theſe they compoſed various figures, of which they demonſtrated many properties, and reſolved divers problems concerning them. Theſe problems they attempted always to reſolve by the deſcribing ſtraight lines and circles. For inſtance, let a ſquare A B C D (fig. 61.) be propoſed, and let it be required to make another

[a] See Method. Increment. prop. 25.

ther ſquare in any aſſigned proportion to this. Prolong one ſide, as D A, of this ſquare to E, till A E bear the ſame proportion to A D, as the new ſquare is to bear to the ſquare A C. If the oppoſite ſide B C of the ſquare A C be alſo prolonged to F, till B F be equal to A E, and E F be afterwards drawn, I ſuppoſe my readers will eaſily conceive, that the figure ABFE will bear to the ſquare A B C D the ſame proportion, as the line A E bears to the line A D. Therefore the figure A B F E will be equal to the new ſquare, which is to be found, but is not it ſelf a ſquare, becauſe the ſide A E is not of the ſame length with the ſide E F. But to find a ſquare equal to the figure A B F E you muſt proceed thus. Divide the line D E into two equal parts in the point G, and to the center G with the interval G D deſcribe the circle D H E I; then prolong the line A B, till it meets the circle in K; and make the ſquare AKLM, which ſquare will be equal to the figure ABFE, and bear to the ſquare A B C D the ſame proportion, as the line A E bears to A D.

78. I SHALL not proceed to the proof of this, having only here ſet it down as a ſpecimen of the method of reſolving geometrical problems by the deſcription of ſtraight lines and circles. But there are ſome problems, which cannot be reſolved by drawing ſtraight lines or circles upon a plane. For the management therefore of theſe they took into conſideration ſolid figures, and of the ſolid figures they found that, which is called a cone, to be the moſt uſeful.

79. A cone is thus defined by Euclide in his elements of geometry [a]. If to the ſtraight line A B (in fig. 62.) another ſtraight line, as A C, be drawn perpendicular, and the two extremities B and C be joined by a third ſtraight line compoſing the triangle A C B (for ſo every figure is called, which is included under three ſtraight lines:) then the two points A and B being held fixed, as two centers, and the triangle A C B being turned round upon the line A B, as on an axis; the line A C will deſcribe a circle, and the figure A C B will deſcribe a cone, of the form repreſented by the figure BCDEF (fig. 63.) in which the circle CDEF is uſually called the baſe of the cone, and B the vertex.

80. Now by this figure may ſeveral problems be reſolved, which cannot by the ſimple deſcription of ſtraight lines and circles upon a plane. Suppoſe for inſtance, it were required to make a cube, which ſhould bear any aſſigned proportion to ſome other cube named. I need not here inform my readers, that a cube is the figure of a dye. This problem was much celebrated among the ancients, and was once inforced by the command of an oracle. This problem may be performed by a cone thus. Firſt make a cone from a triangle, whoſe ſide A C ſhall be half the length of the ſide B C. Then on the plane A B C D (fig. 64.) let the line E F be exhibited equal in length to the ſide of the cube propoſed; and let the line F G be drawn perpendicular to E F, and of ſuch a length, that it bear the ſame proportion to E F, as the

[a] Lib. XI. Def.

cube

cube to be ſought is required to bear to the cube propoſed. Through the points E, F, and G let the circle F H I be deſcribed. Then let the line E F be prolonged beyond F to K, that F K be equal to F E, and let the triangle F K L, having all its ſides F K, K L, L F equal to each other, be hung down perpendicularly from the plane A B C D. After this, let another plane M N O P be extended through the point L, ſo as to be equidiſtant from the former plane A B C D, and in this plane let the line Q L R be drawn ſo, as to be equidiſtant from the line E F K. All this being thus prepared, let ſuch a cone, as was above directed to be made, be ſo applied to the plane M N O P, that it touch this plane upon the line Q R, and that the vertex of the cone be applied to the point L. This cone, by cutting through the firſt plane A B C D, will croſs the circle F H I before deſcribed. And if from the point S, where the ſurface of this cone interſects the circle, the line S T be drawn ſo, as to be equidiſtant from the line E F; the line F T will be equal to the ſide of the cube ſought: that is, if there be two cubes or dyes formed, the ſide of one being equal to E F, and the ſide of the other equal to F T; the former of theſe cubes ſhall bear the ſame proportion to the latter, as the line E F bears to F G.

81. INDEED this placing a cone to cut through a plane is not a practicable method of reſolving problems. But when the geometers had diſcovered this uſe of the cone, they applied themſelves to conſider the nature of the lines, which will be produced by the interſection of the ſurface of a cone

and a plane; whereby they might be enabled both to reduce these kinds of solutions to practice, and also to render their demonstrations concise and elegant.

82. Whenever the plane, which cuts the cone, is equidistant from another plane, that touches the cone on the side; (which is the case of the present figure;) the line, wherein the plane cuts the surface of the cone, is called a parabola. But if the plane, which cuts the cone, be so inclined to this other, that it will pass quite through the cone (as in fig. 65.) such a plane by cutting the cone produces the figure called an ellipsis, in which we shall hereafter shew the earth and other planets to move round the sun. If the plane, which cuts the cone, recline the other way (as in fig. 66.) so as not to be parallel to any plane, whereon the cone can lie, nor yet to cut quite through the cone; such a plane shall produce in the cone a third kind of line, which is called an hyperbola. But it is the first of these lines named the parabola, wherein bodies, that are thrown obliquely, will be carried by the force of gravity; as I shall here proceed to shew, after having first directed my readers how to describe this sort of line upon a plane, by which the form of it may be seen.

83. To any straight line AB (fig. 67.) let a straight ruler CD be so applied, as to stand against it perpendicularly. Upon the edge of this ruler let another ruler EF be so placed, as to move along upon the edge of the first ruler CD, and keep always perpendicular to it. This being so disposed, let any point, as G, be taken in the line AB, and let a string equal

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in length to the ruler EF be faſtened by one end to the point G, and by the other to the extremity F of the ruler EF. Then if the ſtring be held down to the ruler EF by a pin H, as is repreſented in the figure; the point of this pin, while the ruler EF moves on the ruler CD, ſhall deſcribe the line IKL, which will be one part of the curve line, whoſe deſcription we were here to teach: and by applying the rulers in the like manner on the other ſide of the line AB, we may deſcribe the other part IM of this line. If the diſtance CG be equal to half the line EF in fig. 64, the line MIL will be that very line, wherein the plane ABCD in that figure cuts the cone.

84. THE line AI is called the axis of the parabola MIL, and the point G is called the focus.

85. Now by comparing the effects of gravity upon falling bodies, with what is demonſtrated of this figure by the geometers, it is proved, that every body thrown obliquely is carried forward in one of theſe lines, the axis whereof is perpendicular to the horizon.

86. THE geometers demonſtrate, that if a line be drawn to touch a parabola in any point, as the line AB (in fig. 68.) touches the parabola CD, whoſe axis is YZ, in the point E; and ſeveral lines FG, HI, KL be drawn parallel to the axis of the parabola: then the line FG will be to HI in the duplicate proportion of EF to EH, and FG to KL in the duplicate proportion of EF to EK; likewiſe HI to KL in the duplicate proportion of EH to EK. What is to be underſtood by duplicate or two-fold

proportion, has been already explained[a]. Accordingly I mean here, that if the line M be taken to bear the ſame proportion to EH, as EH bears to EF, HI will bear the ſame proportion to FG, as M bears to EF; and if the line N bears the ſame proportion to EK, as EK bears to EF, KL will bear the ſame proportion to FG, as N bears to EF; or if the line O bear the ſame proportion to EK, as EK bears to EH, KL will bear the ſame proportion to HI, as O bears to EH.

87. THIS property is eſſential to the parabola, being ſo connected with the nature of the figure, that every line poſſeſſing this property is to be called by this name.

88. NOW ſuppoſe a body to be thrown from the point A (in fig. 69.) towards B in the direction of the line AB. This body, if left to it ſelf, would move on with a uniform motion through this line AB. Suppoſe the eye of a ſpectator to be placed at the point C juſt under the point A; and let us imagine the earth to be ſo put into motion along with the body, as to carry the ſpectator's eye along the line CD parallel to AB; and that the eye ſhould move on with the ſame velocity, wherewith the body would proceed in the line AB, if it were to be left to move without any diſturbance from its gravitation towards the earth. In this caſe if the body moved on without being drawn towards the earth, it would appear to the ſpectator to be at reſt. But if the power of gravity exerted it ſelf on the body, it would appear to the ſpe-

[a] Chap. 2. § 17.

ctator

ctator to fall directly down. Suppose at the distance of time, wherein the body by its own progressive motion would have moved from A to E, it should appear to the spectator to have fallen through a length equal to EF: then the body at the end of this time will actually have arrived at the point F. If in the space of time, wherein the body would have moved by its progressive motion from A to G, it would have appeared to the spectator to have fallen down the space GH: then the body at the end of this greater interval of time will be arrived at the point H. Now if the line AFHI be that, through which the body actually passes; from what has here been said, it will follow, that this line is one of those, which I have been describing under the name of the parabola. For the distances EF, GH, through which the body is seen to fall, will increase in the duplicate proportion of the times [a]; but the lines AE, AG will be proportional to the times wherein they would have been described by the single progressive motion of the body: therefore the lines EF, GH will be in the duplicate proportion of the lines AF, AG; and the line AFHI possesses the property of the parabola.

89. IF the earth be not supposed to move along with the body, the case will be a little different. For the body being constantly drawn directly towards the center of the earth, the body in its motion will be drawn in a direction a little oblique to that, wherein it would be drawn by the earth in motion, as before supposed. But the distance to the center of the

[a] See above Ch. 2. § 17.

earth

earth bears ſo vaſt a proportion to the greateſt length, to which we can throw bodies, that this obliquity does not merit any regard. From the ſequel of this diſcourſe it may indeed be collected, what line the body being thrown thus would be found to deſcribe, allowance being made for this obliquity of the earth's action[a]. This is the diſcovery of Sir Is. Newton; but has no uſe in this place. Here it is abundantly ſufficient to conſider the body as moving in a parabola.

90. The line, which a projected body deſcribes, being thus known, practical methods have been deduced from hence for directing the ſhot of great guns to ſtrike any object deſired. This work was firſt attempted by Galileo, and ſoon after farther improved by his ſcholar Torricelli; but has lately been rendred more complete by the great Mr. Cotes, whoſe immature death is an unſpeakable loſs to mathematical learning. If it be required to throw a body from the point A (in fig. 70.) ſo as to ſtrike the point B; through the points A, B draw the ſtraight line CD, and erect the line AE perpendicular to the horizon, and of four times the height, from which a body muſt fall to acquire the velocity, wherewith the body is intended to be thrown. Through the points A and E deſcribe a circle, that ſhall touch the line CD in the point A. Then from the point B draw the line BF perpendicular to the horizon, interſecting the circle in the points G and H. This being done, if the body be projected directly towards either of theſe points G or H, it ſhall fall upon the point B; but with this difference, that, if it be thrown in

[a] From B. II. Ch. 3.

in the direction A G, it ſhall ſooner arrive at B, than if it were projected in the direction A H. When the body is projected in the direction A G; the time, it will take up in arriving at B, will bear the ſame proportion to the time, wherein it would fall down through one fourth part of A E, as A G bears to half A E. But when the body is thrown in the direction of A H, the time of its paſſing to B will bear the ſame proportion to the time, wherein it would fall through one fourth part of A E, as A H bears to half A E.

91. IF the line A I be drawn ſo as to divide the angle under E A D in the middle, and the line I K be drawn perpendicular to the horizon; this line will touch the circle in the point I, and if the body be thrown in the direction A I, it will fall upon the point K: and this point K is the fartheſt point in the line A D, which the body can be made to ſtrike, without increaſing its velocity.

92. THE velocity, wherewith the body every where moves, may be found thus. Suppoſe the body to move in the parabola A B (fig. 71.) Erect A C perpendicular to the horizon, and equal to the height, from which a body muſt fall to acquire the velocity, wherewith the body ſets out from A. If you take any points as D and E in the parabola, and draw D F and E G parallel to the horizon; the velocity of the body in D will be equal to what a body will acquire in falling down by its own weight through C F, and in E the velocity will be the ſame, as would be acquired in falling through C G. Thus the body moves ſloweſt at the higheſt point H of the parabola; and at equal diſtances from this point will

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move with equal ſwiftneſs, and deſcend from that higheſt point through the line HB altogether like to the line AH in which it aſcended; abating only the reſiſtance of the air, which is not here conſidered. If the line HI be drawn from the higheſt point H parallel to the horizon, AI will be equal to $\frac{1}{4}$ of BG in fig. 70, when the body is projected in the direction AG, and equal to $\frac{1}{4}$ of BH, when the body is thrown in the direction AH provided AD be drawn horizontally.

93. THUS I have recounted the principal diſcoveries, which had been made concerning the motion of bodies by Sir ISAAC NEWTON's predeceſſors; all theſe diſcoveries, by being found to agree with experience, contributing to eſtabliſh the laws of motion, from whence they were deduced. I ſhall therefore here finiſh what I had to ſay upon thoſe laws; and conclude this chapter with a few words concerning the diſtinction which ought to be made between abſolute and relative motion. For ſome have thought fit to confound them together; becauſe they obſerve the laws of motion to take place here on the earth, which is in motion, after the ſame manner as if it were at reſt. But Sir ISAAC NEWTON has been careful to diſtinguiſh between the relative and abſolute conſideration both of motion and time[a]. The aſtronomers anciently found it neceſſary to make this diſtinction in time. Time conſidered in it ſelf paſſes on equably without relation to any thing external, being the proper meaſure of the continuance and duration of all things. But it is moſt frequently conceived of by us under a relative view to ſome ſucceſſion in

[a] Prin. Philoſ. pag. 7, &c.

ſenſible things, of which we take cognizance. The ſucceſ-ſion of the thoughts in our own minds is that, from whence we receive our firſt idea of time, but is a very uncertain meaſure thereof; for the thoughts of ſome men flow on much more ſwiftly, than the thoughts of others; nor does the ſame perſon think equally quick at all times. The motions of the heavenly bodies are more regular; and the eminent diviſion of time into night and day, made by the ſun, leads us to meaſure our time by the motion of that luminary: nor do we in the affairs of life concern our ſelves with any inequality, which there may be in that motion; but the ſpace of time which comprehends a day and night is rather ſuppoſed to be always the ſame. However aſtronomers anciently found theſe ſpaces of time not to be always of the ſame length, and have taught how to compute their differences. Now the time, when ſo equated as to be rendered perfectly equal, is the true meaſure of duration, the other not. And therefore this latter, which is abſolutely true time, differs from the other, which is only apparent. And as we ordinarily make no diſtinction between apparent time, as meaſured by the ſun, and the true; ſo we often do not diſtinguiſh in our uſual diſcourſe between the real, and the apparent or relative motion of bodies; but uſe the ſame words for one, as we ſhould for the other. Though all things about us are really in motion with the earth; as this motion is not viſible, we ſpeak of the motion of every thing we ſee, as if our ſelves and the earth ſtood ſtill. And even in other caſes, where we diſcern the motion of bodies, we often ſpeak of them not in relation to the whole motion we ſee, but with regard to other

bodies, to which they are contiguous. If any body were lying on a table; when that table ſhall be carried along, we ſay the body reſts upon the table, or perhaps abſolutely, that the body is at reſt. However philoſophers muſt not reject all diſtinction between true and apparent motions, any more than aſtronomers do the diſtinction between true and vulgar time; for there is as real a difference between them, as will appear by the following conſideration. Suppoſe all the bodies of the univerſe to have their courſes ſtopped, and reduced to perfect reſt. Then ſuppoſe their preſent motions to be again reſtored; this cannot be done without an actual impreſſion made upon ſome of them at leaſt. If any of them be left untouched, they will retain their former ſtate, that is, ſtill remain at reſt; but the other bodies, which are wrought upon, will have changed their former ſtate of reſt, for the contrary ſtate of motion. Let us now ſuppoſe the bodies left at reſt to be annihilated, this will make no alteration in the ſtate of the moving bodies; but the effect of the impreſſion, which was made upon them, will ſtill ſubſiſt. This ſhews the motion they received to be an abſolute thing, and to have no neceſſary dependence upon the relation which the body ſaid to be in motion has to any other body [a].

94. Besides abſolute and relative motion are diſtinguiſhable by their Effects. One effect of motion is, that bodies, when moved round any center or axis, acquire a certain

[a] See Newton. princip. philoſ. pag. 9. lin. 30.

power,

power, by which they forcibly preſs themſelves from that center or axis of motion. As when a body is whirled about in a ſling, the body preſſes againſt the ſling, and is ready to fly out as ſoon as liberty is given it. And this power is proportional to the true, not relative motion of the body round ſuch a center or axis. Of this Sir ISAAC NEWTON gives the following inſtance [a]. If a pail or ſuch like veſſel near full of water be ſuſpended by a ſtring of ſufficient length, and be turned about till the ſtring be hard twiſted. If then as ſoon as the veſſel and water in it are become ſtill and at reſt, the veſſel be nimbly turned about the contrary way the ſtring was twiſted, the veſſel by the ſtrings untwiſting it ſelf ſhall continue its motion a long time. And when the veſſel firſt begins to turn, the water in it ſhall receive little or nothing of the motion of the veſſel, but by degrees ſhall receive a communication of motion, till at laſt it ſhall move round as ſwiftly as the veſſel it ſelf. Now the definition of motion, which DES CARTES has given us upon this principle of making all motion meerly relative, is this: that motion, is a removal of any body from its vicinity to other bodies, which were in immediate contact with it, and are conſidered as at reſt [b]. And if this be compared with what he ſoon after ſays, that there is nothing real or poſitive in the body moved, for the ſake of which we aſcribe motion to it, which is not to be found as well in the contiguous bodies, which are conſidered as at reſt [c]; it will follow from thence, that we may conſider the veſſel as at reſt

[a] Princip. Philoſ. pag. 10.
[b] Renat. Des Cart. Princ. Philoſ. Part. II. § 25.
[c] Ibid. § 30.

 and

and the water as moving in it: and the water in reſpect of the veſſel has the greateſt motion, when the veſſel firſt begins to turn, and loſes this relative motion more and more, till at length it quite ceaſes. But now, when the veſſel firſt begins to turn, the ſurface of the water remains ſmooth and flat, as before the veſſel began to move; but as the motion of the veſſel communicates by degrees motion to the water, the ſurface of the water will be obſerved to change, the water ſubſiding in the middle and riſing at the edges: which elevation of the water is cauſed by the parts of it preſſing from the axis, they move about; and therefore this force of receding from the axis of motion depends not upon the relative motion of the water within the veſſel, but on its abſolute motion; for it is leaſt, when that relative motion is greateſt, and greateſt, when that relative motion is leaſt, or none at all.

95. THUS the true cauſe of what appears in the ſurface of this water cannot be aſſigned, without conſidering the water's motion within the veſſel. So alſo in the ſyſtem of the world, in order to find out the cauſe of the planetary motions, we muſt know more of the real motions, which belong to each planet, than is abſolutely neceſſary for the uſes of aſtronomy. If the aſtronomer ſhould ſuppoſe the earth to ſtand ſtill, he could aſcribe ſuch motions to the celeſtial bodies, as ſhould anſwer all the appearances; though he would not account for them in ſo ſimple a manner, as by attributing motion to the earth. But the motion of the earth muſt of neceſſity be conſidered, before the real cauſes, which actuate the planetary ſyſtem, can be diſcovered.

CHAP.

CHAP. III.

Of CENTRIPETAL FORCES.

WE have juſt been deſcribing in the preceding chapter the effects produced on a body in motion, from its being continually acted upon by a power always equal in ſtrength, and operating in parallel directions [a]. But bodies may be acted upon by powers, which in different places ſhall have different degrees of force, and whoſe ſeveral directions ſhall be variouſly inclined to each other. The moſt ſimple of theſe in reſpect to direction is, when the power is pointed conſtantly to one center. This is truly the caſe of that power, whoſe effects we deſcribed in the foregoing chapter; though the center of that power is ſo far removed, that the ſubject then before us is moſt conveniently to be conſidered in the light, wherein we have placed it: But Sir ISAAC NEWTON has conſidered very particularly this other caſe of powers, which are conſtantly directed to the ſame center. It is upon this foundation, that all his diſcoveries in the ſyſtem of the world are raiſed. And therefore, as this ſubject bears ſo very great a ſhare in the philoſophy, of which I am diſcourſing, I think it proper in this place to take a ſhort view of ſome of the general effects of theſe powers, before we come to apply them particularly to the ſyſtem of the world.

[a] § 85, &c.

2. THESE

2. THESE powers or forces are by Sir ISAAC NEWTON called centripetal; and their firſt effect is to cauſe the body, on which they act, to quit the ſtraight courſe, wherein it would proceed if undiſturbed, and to deſcribe an incurvated line, which ſhall always be bent towards the center of the force. It is not neceſſary, that ſuch a power ſhould cauſe the body to approach that center. The body may continue to recede from the center of the power, notwithſtanding its being drawn by the power; but this property muſt always belong to its motion, that the line, in which it moves, will continually be concave towards the center, to which the power is directed. Suppoſe A (in fig. 72.) to be the center of a force. Let a body in B be moving in the direction of the ſtraight line BC, in which line it would continue to move, if undiſturbed; but being attracted by the centripetal force towards A, the body muſt neceſſarily depart from this line BC, and being drawn into the curve line BD, muſt paſs between the lines AB and BC. It is evident therefore, that the body in B being gradually turned off from the ſtraight line BC, it will at firſt be convex toward the line BC, and conſequently concave towards the point A: for theſe centripetal powers are ſuppoſed to be in ſtrength proportional to the power of gravity, and, like that, not to be able after the manner of an impulſe to turn the body ſenſibly out of its courſe into a different one in an inſtant, but to take up ſome ſpace of time in producing a viſible effect. That the curve will always continue to have its concavity towards A may thus appear. In the line BC near to B take any point as E, from which the line EFG may be ſo drawn

drawn, as to touch the curve line B D in ſome point as F. Now when the body is come to F, if the centripetal power were immediately to be ſuſpended, the body would no longer continue to move in a curve line, but being left to it ſelf would forthwith reaſſume a ſtraight courſe; and that ſtraight courſe would be in the line F G: for that line is in the direction of the body's motion at the point F. But the centripetal force continuing its energy, the body will be gradually drawn from this line F G ſo as to keep in the line F D, and make that line near the point F to be convex toward F G, and concave toward A. After the ſame manner the body may be followed on in its courſe through the line B D, and every part of that line be ſhewn to be concave toward the point A.

3. THIS then is the conſtant character belonging to thoſe motions, which are carried on by centripetal forces; that the line, wherein the body moves, is throughout concave towards the center of the force. In reſpect to the ſucceſſive diſtances of the body from the center there is no general rule to be laid down; for the diſtance of the body from the center may either increaſe, or decreaſe, or even keep always the ſame. The point A (in fig. 73.) being the center of a centripetal force, let a body at B ſet out in the direction of the ſtraight line B C perpendicular to the line A B drawn from A to B. It will be eaſily conceived, that there is no other point in the line B C ſo near to A, as the point B; that A B is the ſhorteſt of all the lines, which can be drawn from A to any part of the line B C; all other lines, as A D, or A E, drawn from A to the line B C being longer than A B. Hence it follows, that the body ſet-

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ting out from B, if it moved in the line BC, it would recede more and more from the point A. Now as the operation of a centripetal force is to draw a body towards the center of the force: if ſuch a force act upon a reſting body, it muſt neceſſarily put that body ſo into motion, as to cauſe it to move towards the center of the force: if the body were of it ſelf moving towards that center, the centripetal force would accelerate that motion, and cauſe it to move faſter down: but if the body were in ſuch a motion, as being left to itſelf it would recede from this center, it is not neceſſary, that the action of a centripetal power upon it ſhould immediately compel the body to approach the center, from which it would otherwiſe have receded; the centripetal power is not without effect, if it cauſe the body to recede more ſlowly from that center, than otherwiſe it would have done. Thus in the caſe before us, the ſmalleſt centripetal power, if it act on the body, will force it out of the line BC, and cauſe it to paſs in a bent line between BC and the point A, as has been before explained. When the body, for inſtance, has advanced to the line AD, the effect of the centripetal force diſcovers it ſelf by having removed the body out of the line BC, and brought it to croſs the line AD ſomewhere between A and D: ſuppoſe at F. Now AD being longer than AB, AF may alſo be longer than AB. The centripetal power may indeed be ſo ſtrong, that AF ſhall be ſhorter than AB; or it may be ſo evenly balanced with the progreſſive motion of the body, that AF and AB ſhall be juſt equal: and in this laſt caſe, when the centripetal force is of that ſtrength, as conſtantly to draw the body as much toward

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the center, as the progreſſive motion would carry it off, the body will deſcribe a circle about the center A, this center of the force being alſo the center of the circle.

4. IF the body, inſtead of ſetting out in the line BC perpendicular to AB, had ſet out in another line BG more inclined towards the ~~line AB~~, moving in the curve line BH; then as the body, if it were to continue its motion in the line BG, would for ſome time approach the center A; the centripetal force would cauſe it to make greater advances toward that center. But if the body were to ſet out in the line BI reclined the other way from the perpendicular BC, and were to be drawn by the centripetal force into the curve line BK; the body, notwithſtanding any centripetal force, would for ſome time recede from the center; ſince ſome part at leaſt of the curve ~~line BK lies between~~ the line BI and the perpendicular BC.

5. THUS far we have explained ſuch effects, as attend every centripetal force. But as theſe forces may be very different in regard to the different degrees of ſtrength, wherewith they act upon bodies in different places; I ſhall now proceed to make mention in general of ſome of the differences attending theſe centripetal motions.

6. TO reaſſume the conſideration of the laſt mentioned caſe. Suppoſe a centripetal power directed toward the point A (in fig. 74.) to act on a body in B, which is moving in the direction of the ſtraight line BC, the line BC reclining off from AB. If from A the ſtraight lines AD, AE, AF are

drawn at pleaſure to the line CB; the line CB being prolonged beyond B to G, it appears that AD is inclined to the line GC more obliquely, than AB is inclined to it, AE is inclined more obliquely than AD, and AF more than AE. To ſpeak more correctly, the angle under ADG is leſs than that under ABG, the angle under AEG leſs than that under ADG, and the angle under AFG leſs than that under AEG. Now ſuppoſe the body to move in the curve line BHIK. Then it is here likewiſe evident, that the line BHIK being concave towards A, and convex towards the line BC, it is more and more turned off from the line BC; ſo that in the point H the line AH will be leſs obliquely inclined to the curve line BHIK, than the ſame line AHD is inclined to BC at the point D; at the point I the inclination of the line AI to the curve line will be more different from the inclination of the ſame line AIE to the line BC at the point E; and in the points K and F the difference of inclination will be ſtill greater; and in both the inclination at the curve will be leſs oblique, than at the ſtraight line BC. But the ſtraight line AB is leſs obliquely inclined to BG, than AD is inclined towards DG: therefore although the line AH be leſs obliquely inclined towards the curve HB, than the ſame line AHD is inclined towards DG; yet it is poſſible, that the inclination at H may be more oblique, than the inclination at B. The inclination at H may indeed be leſs oblique than the other, or they may be both the ſame. This depends upon the degree of ſtrength, wherewith the centripetal force exerts it ſelf, during the paſſage of the body from B to H. After the ſame manner the inclinations at I and K depend entirely on the degree

gree of ſtrength, wherewith the centripetal force acts on the body in its paſſage from H to K: if the centripetal force be weak enough, the lines A H and A I drawn from the center A to the body at H and at I ſhall be more obliquely inclined to the curve, than the line A B is inclined towards B G. The centripetal force may be of that ſtrength as to render all theſe inclinations equal, or if ſtronger, the inclinations at I and K will be leſs oblique than at B. Sir ISAAC NEWTON has particularly ſhewn, that if the centripetal power decreaſes after a certain manner with the increaſe of diſtance, a body may deſcribe ſuch a curve line, that all the lines drawn from the center to the body ſhall be equally inclined to that curve line [a]. But I do not here enter into any particulars, my preſent intention being only to ſhew, that it is poſſible for a body to be acted upon by a force continually drawing it down towards a center, and yet that the body ſhall continue to recede from that center; for here as long as the lines A H, A I, &c drawn from the center A to the body do not become leſs oblique to the curve, in which the body moves; ſo long ſhall thoſe lines perpetually increaſe, and conſequently the body ſhall more and more recede from the center.

7. BUT we may obſerve farther, that if the centripetal power, while the body increaſes its diſtance from the center, retain ſufficient ſtrength to make the lines drawn from the center to the body to become at length leſs oblique to the curve; then if this diminution of the obliquity continue, till

[a] Princip. Philoſ. Lib. I. prop. 9.

at laſt the line drawn from the center to the body ſhall ceaſe to be obliquely inclined to the curve, and ſhall become perpendicular thereto; from this inſtant the body ſhall no longer recede from the center, but in its following motion it ſhall again deſcend, and ſhall deſcribe a curve line in all reſpects like to that, which it has deſcribed already; provided the centripetal power, every where at the ſame diſtance from the center, acts with the ſame ſtrength. So we obſerved in the preceding chapter, that, when the motion of a projectile became parallel to the horizon, the projectile no longer aſcended, but forthwith directed its courſe downwards, deſcending in a line altogether like that, wherein it had before aſcended [a].

8. THIS return of the body may be proved by the following propoſition: that if the body in any place, ſuppoſe at I, were to be ſtopt, and be thrown directly backward with the velocity, wherewith it was moving forward in that point I; then the body, by the action of the centripetal force upon it, would move back again over the path I H B, in which it had before advanced forward, and would arrive again at the point B in the ſame ſpace of time, as was taken up in its paſſage from B to I; the velocity of the body at its return to the point B being the ſame, as that wherewith it firſt ſet out from that point. To give a full demonſtration of this propoſition, would require that uſe of mathematics, which I here purpoſe to avoid; but, I believe, it will appear in great meaſure evident from the following conſiderations.

[a] § 92.

9. SUP-

9. SUPPOSE (in fig. 75.) that a body were carried after the following manner through the bent figure ABCDEF, compoſed of the ſtraight lines AB, BC, CD, DE, EF. Firſt let it be moving in the line AB, from A towards B, with any uniform velocity. At B let the body receive an impulſe directed toward ſome point, as G, taken within the concavity of the figure. ~~Now whereas~~ this body, when once moving in the ſtraight line AB, will continue to move on in this line, ſo long as it ſhall be left to it ſelf; but being diſturbed at the point B in its motion by the impulſe, which there acts upon it, it will be turned out of this line AB into ſome other ſtraight line, wherein it will afterwards continue to move, as long as it ſhall be left to itſelf. Therefore let this impulſe have ſtrength ſufficient to turn the body into the line BC. Then let the body move on undiſturbed from B to C, but at C let it receive another impulſe pointed toward the ſame point G, and of ſufficient ſtrength to turn the body into the line CD. At D let a third impulſe, directed like the reſt to the point G, turn the body into the line DE. And at E let another impulſe, directed likewiſe to the point G, turn the body into the line EF. Now, I ſay, if the body while moving in the line EF be ſtopt, and turned back again in this line with the ſame velocity, as that wherewith it was moving forward in this line; then by the repetition of the former impulſe at E the body will be turned into the line ED, and move in it from E to D with the ſame velocity as before it moved with from D to E; by the repetition of the impulſe at D, when the body ſhall have returned to that point, it will be turned into the line DC; and by the repetition of the other impulſes at C and B the

the body will be brought back again into the line B A, with the velocity, wherewith it first moved in that line.

10. This I prove as follows. Let D E and F E be continued beyond E. In D E thus continued take at pleasure the length E H, and let H I be so drawn, as to be equidistant from the line G E. Then, by what has been written upon the second law of motion [a], it follows, that after the impulse on the body in E it will move through E I in the same time, as it would have imployed in moving from E to H, with the velocity which it had in the line D E. In F E prolonged take E K equal to E I, and draw K L equidistant from G E. Then, because the body is thrown back in the line F E with the same velocity as that wherewith it went forward in that line; if, when the body was returned to E, it were permitted to go straight on, it would pass through E K in the same time, as it took up in passing through E I, when it went forward in the line E F. But, if at the body's return to the point E, such an impulse directed toward the point D were to be given it, whereby it should be turned into the line D E; I say, that the impulse necessary to produce this effect must be equal to that, which turned the body out of the line D E into E F; and that the velocity, with which the body will return into the line E D, is the same, as that wherewith it before moved through this line from D to E. Because E K is equal to E I, and K L and H I, being each equidistant from G E, are by consequence equidistant from each other; it follows, that the two

[a] Ch. II. § 22.

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triangular figures IEH and KEL are altogether like and equal to each other. If I were writing to mathematicians, I might refer them to some propositions in the elements of EUCLID for the proof of this [a]: but as I do not here address my self to such, so I think this assertion will be evident enough without a proof in form; at least I must desire my readers to receive it as a proposition ~~true in~~ geometry. But these two triangular figures being altogether like each other and equal; as EK is equal to EI, so EL is equal to EH, and KL equal to HI. Now the body after its return to E being turned out of the line FE into ED by an impulse acting upon it in E, after the manner above expressed; the body will receive such a velocity by this impulse, as will carry it through EL in the same time, as it would have imployed in passing through EK, if it had gone on in that line undisturbed. And it has already been observed, that the time, in which the body would pass over EK with the velocity wherewith it returns, is equal to the time it took up in going forward from E to I; that is, equal to the time, in which it would have gone through EH with the velocity, wherewith it moved from D to E. Therefore the time, in which the body will pass through EL after its return into the line ED, is the same, as would have been taken up by the body in passing through EH with the velocity, wherewith the body first moved in the line DE. Since therefore EL and EH are equal, the body returns into the line DE with the velocity, which it had before in that line. Again I say, the second impulse in E is equal to the first. By what has

[a] Viz. L. I. prop. 30, 29, & 26.

been

been ſaid on the ſecond law of motion concerning the effect of oblique impulſes[a], it will be underſtood, that the impulſe in E, whereby the body was turned out of the line DE into the line EF, is of ſuch ſtrength, that if the body had been at reſt, when this impulſe had acted upon it, this impulſe would have communicated ſo much motion to the body, as would have carried it through a length equal to HI, in the time wherein the body would have paſſed from E to H, or in the time wherein it paſſed from E to I. In the ſame manner, on the return of the body, the impulſe in E, whereby the body is turned out of the line FE into ED, is of ſuch ſtrength, that if it had acted on the body at reſt, it would have cauſed the body to move through a length equal to KL, in the ſame time, as the body would imploy in paſſing through EK with the velocity, wherewith it returns in the line FE. Therefore the ſecond impulſe, had it acted on the body at reſt, would have cauſed it to move through a length equal to KL in the ſame ſpace of time, as would be taken up by the body in paſſing through a length equal to HI, were the firſt impulſe to act on the body when at reſt. That is, the effects of the firſt and ſecond impulſe on the body when at reſt would be the ſame; for KL and HI are equal: conſequently the ſecond impulſe is equal to the firſt.

11. THUS if the body be returned through FE with the velocity, wherewith it moved forward; we have ſhewn how by the repetition of the impulſe, which acted on it at E, the

[a] Ch. II. §21, 22.

body

body will return again into the line DE with the velocity, which it had before in that line. By the ſame proceſs of reaſoning it may be proved, that, when the body is returned back to D, the impulſe, which before acted on the body at that point, will throw the body into the line DC with the velocity, which it firſt had in that line; and the other impulſes being ſucceſſively repeated, the body will at length be brought back again into the line BA with the velocity, wherewith it ſet out in that line.

12. THUS theſe impulſes, by acting over again in an inverted order all their operation on the body, bring it back again through the path, in which it had proceeded forward. And this obtains equally, whatever be the number of the ſtraight lines, whereof this curve figure is compoſed. Now by a method of reaſoning, which Sir ISAAC NEWTON makes great uſe of, and which he introduced into geometry, thereby greatly inriching that ſcience [a]; we might make a tranſition from this figure compoſed of a number of ſtraight lines to a figure of one continued curvature, and from a number of ſeparate impulſes repeated at diſtinct intervals to a continual centripetal force, and ſhew, that, becauſe what has been here advanced holds univerſally true, whatever be the number of ſtraight lines, whereof the curve figure ACF is compoſed, and howſoever frequently the impulſes at the angles of this figure are repeated; therefore the ſame will ſtill remain true, although this figure ſhould be converted into one of a continued curvature, and theſe diſtinct impulſes ſhould be

[a] viz. His doctrine of prime and ultimate ratios.

changed into a continual centripetal force. But as the explaining this method of reaſoning is foreign to my preſent deſign; ſo I hope my readers, after what has been ſaid, will find no difficulty in receiving the propoſition laid down above: that, if the body, which has moved through the curve line B H I (in fig. 74.) from B to I, when it is come to I, be thrown directly back with the ſame velocity as that, wherewith it proceeded forward, the centripetal force, by acting over again all its operation on the body, ſhall bring the body back again in the line I H B: and as the motion of the body in its courſe from B to I was every where in ſuch a manner oblique to the line drawn from the center to the body, that the centripetal power acted in ſome degree againſt the body's motion, and gradually diminiſhed it; ſo in the return of the body, the centripetal power will every where draw the body forward, and accelerate its motion by the ſame degrees, as before it retarded it.

13. This being agreed, ſuppoſe the body in K to have the line A K no longer obliquely inclined to its motion. In this caſe, if the body be turned back, in the manner we have been conſidering, it muſt be directed back perpendicularly to A K. But if it had proceeded forward, it would likewiſe have moved in a direction perpendicular to A K; conſequently, whether it move from this point K backward or forward, it muſt deſcribe the ſame kind of courſe. Therefore ſince by being turned back it will go over again the line K I H B; if it be permitted to go forward, the line K L, which it ſhall deſcribe, will be altogether ſimilar to the line K H B.

14. IN like manner we may determine the nature of the motion, if the line, wherein the body ſets out, be inclined (as in fig. 76.) down toward the line BA drawn between the body and the center. If the centripetal power ſo much increaſes in ſtrength, as the body approaches, that it can bend the path, in which the body moves, to that degree, as to cauſe all the lines as ~~AH, AI,~~ AK to remain no leſs oblique to the motion of the body, than AB is oblique to BC; the body ſhall continually more and more approach the center. But if the centripetal power increaſes in ſo much leſs a degree, as to permit the line drawn from the center to the body, as it accompanies the body in its motion, at length to become more and more erect to the curve wherein the body moves, and in the end, ſuppoſe at K, to become perpendicular thereto; from that time the body ſhall riſe again. This is evident from what has been ſaid above; becauſe for the very ſame reaſon here alſo the body ſhall proceed from the point K to deſcribe a line altogether ſimilar to the line, in which it has moved from B to K. Thus, as it was obſerved of the pendulum in the preceding chapter [a], that all the time it approaches towards being perpendicular to the horizon, it more and more deſcends; but, as ſoon as it is come into that perpendicular ſituation, it immediately riſes again by the ſame degrees, as it deſcended by before: ſo here the body more and more approaches the center all the time it is moving from B to K; but thence forward it riſes from the center again by the ſame degrees, as it approached by before.

[a] § 57

15. If (in fig. 77.) the line B C be perpendicular to A B; then it has been obſerved above [a], that the centripetal power may be ſo balanced with the progreſſive motion of the body, that the body may keep moving round the center A conſtantly at the ſame diſtance; as a body does, when whirled about any point, to which it is tyed by a ſtring. If the centripetal power be too weak to produce this effect, the motion of the body will preſently become oblique to the line drawn from itſelf to the center, after the manner of the firſt of the two caſes, which we have been conſidering. If the centripetal power be ſtronger, than what is required to carry the body in a circle, the motion of the body will preſently fall in with the ſecond of the caſes, we have been conſidering.

16. If the centripetal power ſo change with the change of diſtance, that the body, after its motion has become oblique to the line drawn from itſelf to the center, ſhall again become perpendicular thereto; which we have ſhewn to be poſſible in both the caſes treated of above; then the body ſhall in its ſubſequent motion return again to the diſtance of A B, and from that diſtance take a courſe ſimilar to the former: and thus, if the body move in a ſpace free from all reſiſtance, which has been here all along ſuppoſed; it ſhall continue in a perpetual motion about the center, deſcending and aſcending alternately therefrom. If the body ſetting out from B (in fig. 78.) in the line B C perpendicular to A B, deſcribe the line B D E, which in D ſhall be oblique to the line A D, but in E

[a] § 3.

ſhall

ſhall again become erect to AE drawn from the body in E to the center A; then from this point E the body ſhall deſcribe the line EFG altogether like to the line BDE, and at G ſhall be at the ſame diſtance from A, as it was at B. But likewiſe the line AG ſhall be erect to the body's motion. Therefore the body ſhall proceed to deſcribe from G the line GHI altogether ſimilar to ~~the line GF~~E, and at I have the ſame diſtance from the center, as it had at E; and alſo have the line AI erect to its motion: ſo that its following motion muſt be in the line IKL ſimilar to IHG, and the diſtance AL equal to AG. Thus the body will go on in a perpetual round without ceaſing, alternately inlarging and contracting its diſtance from the center.

17. IF it ſo happen, that the point E fall upon the line BA continued beyond A; then the point G will fall on B, I on E, and L alſo on B; ſo that the body will deſcribe in this caſe a ſimple curve line round the center A, like the line BDEF in fig. 79, in which it will continually revolve from B to E and from E to B without end.

18. IF AE in fig. 78 ſhould happen to be perpendicular to AB, in this caſe alſo a ſimple line will be deſcribed; for the point G will fall on the line BA prolonged beyond A, the point I on the line AE prolonged beyond A, and the point L on B.: ſo that the body will deſcribe a line like the curve line BEGI in fig. 80, in which the oppoſite points B and G are equally diſtant from A, and the oppoſite points E and I are alſo equally diſtant from the ſame point A.

19. IN

19. IN other caſes the line deſcribed will have a more complex figure.

20. THUS we have endeavoured to ſhew how a body, while it is conſtantly attracted towards a center, may notwithſtanding by its progreſſive motion keep it ſelf from falling down to that center; but deſcribe ~~about it an~~ endleſs circuit, ſometimes approaching toward that center, and at other times as much receding from the ſame.

21. BUT here we have ſuppoſed, that the centripetal power is of equal ſtrength every where at the ſame diſtance from the center. And this is the caſe of that centripetal power, which will hereafter be ſhewn to be the cauſe, that keeps the planets in their courſes. But a body may be kept on in a perpetual circuit round a center, although the centripetal power have not this property. Indeed a body may by a centripetal force be kept moving in any curve line whatever, that ſhall have its concavity turned every where towards the center of the force.

22. TO make this evident I ſhall firſt propoſe the caſe of a body moving through the incurvated figure ABCDE (in fig. 81.) which is compoſed of the ſtraight lines AB, BC, CD, DE, and EA; the motion being carried on in the following manner. Let the body firſt move in the line AB with any uniform velocity. When it is arrived at the point B, let it receive an impulſe directed toward any point F taken within the figure; and let the impulſe be of that ſtrength as to turn the body out

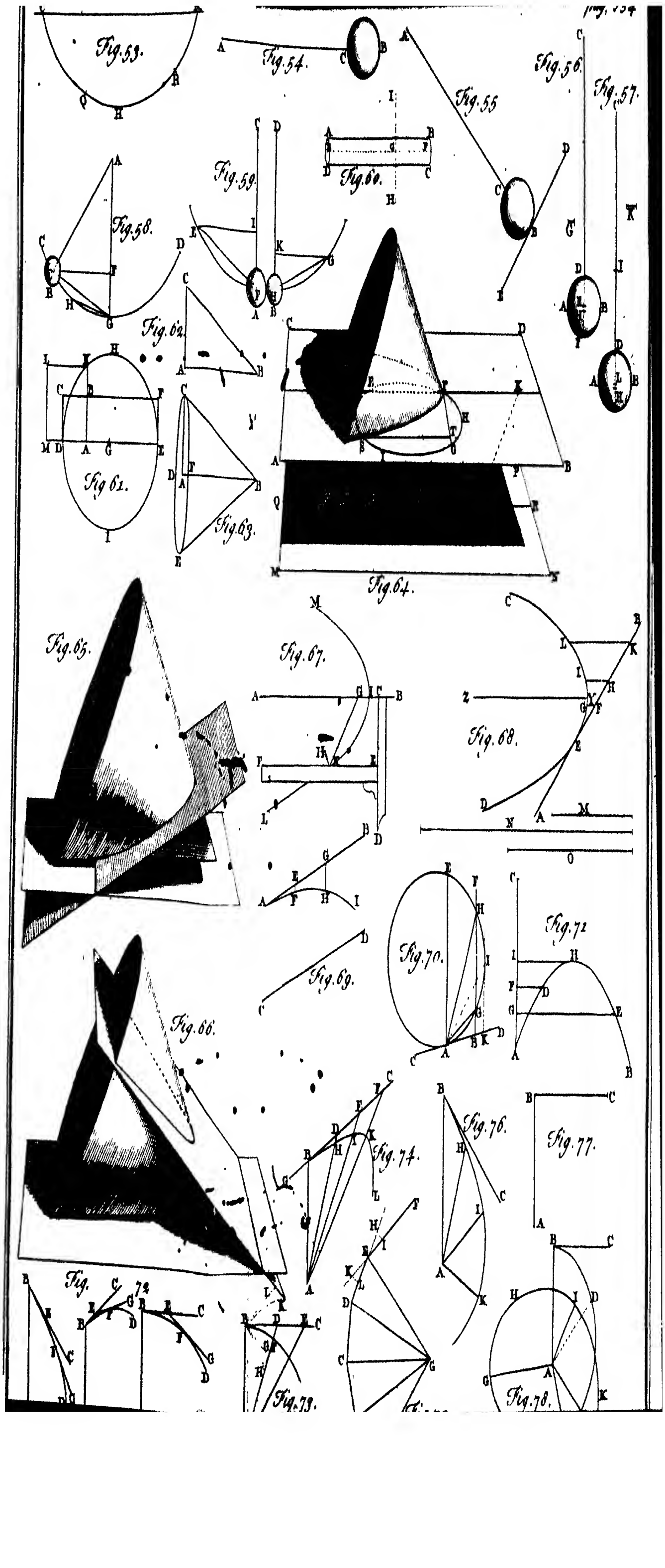
Fig. 53.
Fig. 54.
Fig. 55
Fig. 56.
Fig. 57.
Fig. 58.
Fig. 59.
Fig. 60.
Fig. 61.
Fig. 62.
Fig. 63.
Fig. 64.
Fig. 65.
Fig. 66.
Fig. 67.
Fig. 68.
Fig. 69.
Fig. 70.
Fig. 71
Fig. 72
Fig. 73.
Fig. 74.
Fig. 76.
Fig. 77.
Fig. 78.

of the line AB into the line BC. The body after this impulse, while left to itself, will continue moving in the line BC. At C let the body receive another impulse directed towards the same point F, of such strength, as to turn the body from the line BC into the line CD. At D let the body by another impulse, directed likewise to the point F, be turned out of the line CD into DE. And at E let another impulse, directed toward the point F, turn the body from the line DE into EA. Thus we see how a body may be carried through the figure ABCDE by certain impulses directed always toward the same center, only by their acting on the body at proper intervals, and with due degrees of strength.

23. BUT farther, when the body is come to the point A, if it there receive another impulse directed like the rest toward the point F, and of such a degree of strength as to turn the body into the line AB, wherein it first moved; I say that the body shall return into this line with the same velocity, as it had at first.

24. Let AB be prolonged beyond B at pleasure, suppose to G; and from G let GH be drawn, which if produced should always continue equidistant from BF, or, according to the more usual phrase, let GH be drawn parallel to BF. Then it appears, from what has been said upon the second law of motion[a], that in the time, wherein the body would have moved from B to G, had it not received a new impulse in B, by the means of that impulse it will have acquired a velocity, which will carry it from B to H. After the same manner, if CI be

[a] Ch. 2. §. 22

taken

taken equal to BH, and IK be drawn equidistant from or parallel to CF; the body will have moved from C to K with the velocity, which it has in the line CD, in the same time, as it would have employed in moving from C to I with the velocity, it had in the line BC. Therefore since CI and BH are equal, the body will move through CK in the same time, as it would have taken up in moving from B to G with the original velocity, wherewith it moved through the line AB. Again, DL being taken equal to CK and LM drawn parallel to DF; for the same reason as before the body will move through DM with the velocity, which it has in the line DE, in the same time, as it would imploy in moving through BG with its original velocity. In the last place, if EN be taken equal to DM, and NO be drawn parallel to EF; likewise if AP be taken equal to EO, and PQ be drawn parallel to AF: then the body with the velocity, wherewith it returns into the line AB, will pass through AQ in the same time, as it would have employed in passing through BG with its original velocity. Now as all this follows directly from what has above been delivered, concerning the effect of oblique impulses impressed upon bodies in motion; so we must here observe farther, that it can be proved by geometry, that AQ will always be equal to BG. The proof of this I am obliged, from the nature of my present design, to omit; but this geometrical proposition being granted, it follows, that the body has returned into the line AB with the velocity, which it had, when it first moved in that line; for the velocity, with which it returns into the line AB, will carry it over the line AQ in the same time, as would

have been taken up in its paſſing over an equal line BG with the original velocity.

25. THUS we have found, how a body may be carried round the figure ABCDE by the action of certain impulſes upon it, which ſhould all be pointed toward one center. And we likewiſe ſee, that when the body is brought back again to the point, whence it firſt ſet out; if it there meet with an impulſe ſufficient to turn it again into the line, wherein it moved at firſt, its original velocity will be again reſtored; and by the repetition of the ſame impulſes, the body will be carried again in the ſame round. Therefore if theſe impulſes, which act on the body at the points B, C, D, E, and A, continue always the ſame, the body will make round this figure innumerable revolutions.

26. THE proof, which we have here made uſe of, holds the ſame in any number of ſtraight lines, whereof the figure ABD ſhould be compoſed; and therefore by the method of reaſoning referred to above [a] we are to conclude, that what has here been ſaid upon this rectilinear figure, will remain true, if this figure were changed into one of a continued curvature, and inſtead of diſtinct impulſes acting by intervals at the angles of this figure, we had a continual centripetal force. We have therefore ſhewn, that a body may be carried round in any curve figure ABC (fig. 82.) which ſhall every where be concave towards any one point as D, by the continual action

[a] § 12.

of a centripetal power directed to that point, and when it is returned to the point, from whence it set out, it shall recover again the velocity, with which it departed from that point. It is not indeed always necessary, that it should return again into its first course; for the curve line may have some such figure as the line ABCDBE in fig. 83. In this curve line, if the body set out from B in the direction BF, and moved through the line BCD, till it returned to B; here the body would not enter again into the line BCD, because the two parts BD and BC of the curve line make an angle at the point B: so that the centripetal power, which at the point B could turn the body from the line BF into the curve, will not be able to turn the body into the line BC from the direction, in which it returns to the point B; a forceable impulse must be given the body in the point B to produce that effect.

27. If at the point B, whence the body sets out, the curve line return into it self (as in fig. 82;) then the body, upon its arrival again at B, may return into its former course, and thus make an endless circuit about the center of the centripetal power.

28. What has here been said, I hope, will in some measure enable my readers to form a just idea of the nature of these centripetal motions.

29. I have not attempted to shew, how to find particularly, what kind of centripetal force is necessary to carry a body in any curve line proposed. This is to be deduced from the degree

gree of curvature, which the figure has in each point of it, and requires a long and complex mathematical reaſoning. However I ſhall ſpeak a little to the firſt propoſition, which Sir ISAAC NEWTON lays down for this purpoſe. By this propoſition, when a body is found moving in a curve line, it may be known, whether the body be kept in its courſe by a power always pointed toward the ſame center; and if it be ſo, where that center is placed. The propoſition is this: that if a line be drawn from ſome fixed point to the body, and remaining by one extream united to that point, it be carried round along with the body; then, if the power, whereby the body is kept in its courſe, be always pointed to this fixed point as a center, this line will move over equal ſpaces in equal portions of time. Suppoſe a body were moving through the curve line ABCD (in fig. 84.) and paſſed over the arches AB, BC, CD in equal portions of time; then if a point, as E, can be found, from whence the line EA being drawn to the body in A, and accompanying the body in its motion, it ſhall make the ſpaces EAB, EBC, and ECD equal, over which it paſſes, while the body deſcribes the arches AB, BC, and CD: and if this hold the ſame in all other arches, both great and ſmall, of the curve line ABCD, that theſe ſpaces are always equal, where the times are equal; then is the body kept in this line by a power always pointed to E as a center.

30. THE principle, upon which Sir ISAAC NEWTON has demonſtrated this, requires but ſmall skill in geometry to comprehend. I ſhall therefore take the liberty to cloſe the pre-

ſent chapter with an explication of it; becauſe ſuch an example will give the cleareſt notion of our author's method of applying mathematical reaſoning to theſe philoſophical ſubjects.

31. He reaſons thus. Suppoſe a body ſet out from the point A (in fig. 85.) to move in the ſtraight line AB; and after it had moved for ſome time in that line, it were to receive an impulſe directed to ſome point as C. Let it receive that impulſe at D; and thereby be turned into the line DE; and let the body after this impulſe take the ſame length of time in paſſing from D to E, as it imployed in the paſſing from A to D. Then the ſtraight lines CA, CD, and CE being drawn, Sir Isaac Newton proves, that the and triangular ſpaces CAD and CDE are equal. This he does in the following manner.

32. Let EF be drawn parallel to CD. Then, from what has been ſaid upon the ſecond law of motion [a], it is evident, that ſince the body was moving in the line AB, when it received the impulſe in the direction DC; it will have moved after that impulſe through the line DE in the ſame time, as it would have taken up in moving through DF, provided it had received no diſturbance in D. But the time of the body's moving from D to E is ſuppoſed to be equal to the time of its moving through AD; therefore the time, which the body would have imployed in moving through DF, had it not been diſturbed in D, is equal to the time, wherein it moved through AD: conſequently DF is equal in length to AD; for if the

[a] Ch. 1. ſect. 21, 22.

body

body had gone on to move through the line AB without interruption, it would have moved through all parts thereof with the same velocity, and have passed over equal parts of that line in equal portions of time. Now CF being drawn, since AD and DF are equal, the triangular space CDF is equal to the triangular space CAD. Farther, the line EF being parallel to CD, it is proved[*] by EUCLID, that the triangle CED is equal to the triangle CFD: therefore the triangle CED is equal to the triangle CAD.

33. AFTER the same manner, if the body receive at E another impulse directed toward the point C, and be turned by that impulse into the line EG; if it move afterwards from E to G in the same space of time, as was taken up by its motion from D to E, or from A to D; then CG being drawn, the triangle CEG is equal to CDE. A third impulse at G directed as the two former to C, whereby the body shall be turned into the line GH, will have also the like effect with the rest. If the body move over GH in the same time, as it took up in moving over EG, the triangle CGH will be equal to the triangle CEG. Lastly, if the body at H be turned by a fresh impulse directed toward C into the line HI, and at I by another impulse directed also to C be turned into the line IK; and if the body move over each of the lines HI, and IK in the same time, as it employed in moving over each of the preceding lines AD, DE, EG, and GH: then each of the triangles CHI, and CIK will be equal to each of the preceding. Like-

* Elem. Book I. p. 37.

wise

wiſe as the time, in which the body moves over ADE, is equal to the time of its moving over EGH, and to the time of its moving over HIK; the ſpace CADE will be equal to the ſpace CEGH, and to the ſpace CHIK. In the ſame manner as the time, in which the body moved over ADEG is equal to the time of its moving over GHIK, ſo the ſpace CADEG will be equal to the ſpace CGHIK.

34. FROM this principle Sir ISAAC NEWTON demonſtrates the propoſition mentioned above, by that method of arguing introduced by him into geometry, whereof we have before taken notice [a], by making according to the principles of that method a tranſition from this incurvated figure compoſed of ſtraight lines, to a figure of continued curvature; and by ſhewing, that ſince equal ſpaces are deſcribed in equal times in this preſent figure compoſed of ſtraight lines, the ſame relation between the ſpaces deſcribed and the times of their deſcription will alſo have place in a figure of one continued curvature. He alſo deduces from this propoſition the reverſe of it; and proves, that whenever equal ſpaces are continually deſcribed; the body is acted upon by a centripetal force directed to the center, at which the ſpaces terminate.

[a] § 12.

CHAP.

CHAP. IV.

Of the RESISTANCE of FLUIDS.

BEFORE the cauſe can be diſcovered, which keeps the planets in motion, it is neceſſary firſt to know, whether the ſpace, wherein they move, is empty and void, or filled with any quantity of matter. It has been a prevailing opinion, that all ſpace contains in it matter of ſome kind or other; ſo that where no ſenſible matter is found, there was yet a ſubtle fluid ſubſtance by which the ſpace was filled up; even ſo as to make an abſolute plenitude. In order to examine this opinion, Sir ISAAC NEWTON has largely conſidered the effects of fluids upon bodies moving in them.

2. THESE effects he has reduced under theſe three heads. In the firſt place he ſhews how to determine in what manner the reſiſtance, which bodies ſuffer, when moving in a fluid, gradually increaſes in proportion to the ſpace, they deſcribe in any fluid; to the velocity, with which they deſcribe it; and to the time they have been in motion. Under the ſecond head he conſiders what degree of reſiſtance different bodies moving in the ſame fluid undergo, according to the different proportion between the denſity of the fluid and the denſity of the body. The denſities of bodies, whether fluid or ſolid, are meaſured by the quantity of matter, which is comprehended under the ſame magnitude; that body being the

the moſt denſe or compact, which under the ſame bulk contains the greateſt quantity of ſolid matter, or which weighs moſt, the weight of every body being obſerved above to be proportional to the quantity of matter in it [a]. Thus water is more denſe than cork or wood, iron more denſe than water, and gold than iron. The third particular Sir Is. Newton conſiders concerning the reſiſtance of fluids is the influence, which the diverſity of figure in the reſiſted body has upon its reſiſtance.

3. For the more perfect illuſtration of the firſt of theſe heads, he diſtinctly ſhews the relation between all the particulars ſpecified upon three different ſuppoſitions. The firſt is, that the ſame body be reſiſted more or leſs in the ſimple proportion to its velocity; ſo that if its velocity be doubled, its reſiſtance ſhall become threefold. The ſecond is of the reſiſtance increaſing in the duplicate proportion of the velocity; ſo that, if the velocity of a body be doubled, its reſiſtance ſhall be rendered four times; and if the velocity be trebled, nine times as great as at firſt. But what is to be underſtood by duplicate proportion has been already explained [b]. The third ſuppoſition is, that the reſiſtance increaſes partly in the ſingle proportion of the velocity, ~~and partly in~~ the duplicate proportion thereof.

4. In all theſe ſuppoſitions, bodies are conſidered under two reſpects, either as moving, and oppoſing themſelves

[a] Ch. 1. § 24. [b] Ch. 2. ſect. 17.

againſt

againſt the fluid by that power alone, which is eſſential to them, of reſiſting to the change of their ſtate from reſt to motion, or from motion to reſt, which we have above called their power of inactivity; or elſe, as deſcending or aſcending, and ſo having the power of gravity combined with that other power. Thus our author has ſhewn in all thoſe three ſuppoſitions, in what manner bodies are reſiſted in an uniform fluid, when they move with the aforeſaid progreſſive motion [a]; and what the reſiſtance is, when they aſcend or deſcend perpendicularly [b]. And if a body aſcend or deſcend obliquely, and the reſiſtance be ſingly proportional to the velocity, it is ſhewn how the body is reſiſted in a fluid of an uniform denſity, and what line it will deſcribe [c], which is determined by the meaſurement of the hyperbola, and appears to be no other than that line, firſt conſidered in particular by Dr. BARROW [d], which is now commonly known by the name of the logarithmical curve. In the ſuppoſition that the reſiſtance increaſes in the duplicate proportion of the velocity, our author has not given us the line which would be deſcribed in an uniform fluid; but has inſtead thereof diſcuſſed a problem, which is in ſome ſort the reverſe: to find the denſity of the fluid at all altitudes, by which any given curve line may be deſcribed; which problem is ſo treated by him, as to be applicable to any kind of reſiſtance whatever [e]. But here not unmindful of practice, he ſhews that a body in a fluid of uniform denſity, like the

[a] Newt. Princ. L. II. prop. 2; 5, 6, 7; 11, 12.
[b] [illegible] 8, 9; 13, 14.
[c] Prop. 4.
[d] Prælect. Geometr. pag. 123.
[e] Newton. Princ. Lib. II. prop. 10.

air, will deſcribe a line, which approaches towards an hyperbola; that is, its motion will be nearer to that curve line than to the parabola. And conſequent upon this remark, he ſhews how to determine this hyperbola by experiment, and briefly reſolves the chief of thoſe problems relating to projectiles, which are in uſe in the art of gunnery, in this curve[a]; as TORRICELLI and others have done in the parabola[b], whoſe inventions have been explained at large above[c].

5. OUR author has alſo handled diſtinctly that particular ſort of motion, which is deſcribed by pendulums[d]; and has likewiſe conſidered ſome few caſes of bodies moving in reſiſting fluids round a center, to which they are impelled by a centripetal force, in order to give an idea of thoſe kinds of motions[e].

6. THE treating of the reſiſtance of pendulums has given him an opportunity of inſerting into another part of his work ſome ſpeculations upon the motions of them without reſiſtance, which have a very peculiar elegance; wherein he treats of them as moved by a gravitation acting in the law, which he ſhews to belong to the earth below its ſurface[f]; performing in this kind of gravitation, where the force is proportional to the diſtance from the center, all that HUYGENS had before done in the common ſuppoſition of its being uniform, and acting in parallel lines[g].

[a] Newton. Princ. Lib. II. prop 10. in ſchol.
[b] Torricelli de motu gravium.
[c] Ch. 2. § 85, &c.
[d] Newt. Princ L. II. ſect. 6.
[e] L. II. ſect 4.
[f] See B. II. Ch. 6. § 7. of this treatiſe.
[g] Lib. I. ſect. 10.

7. HUY-

7. HUYGENS at the end of his treatise of the cause of gravity [a] informs us, that he likewise had carried his speculations on the first of these suppositions, of the resistance in fluids being proportional to the velocity of the body, as far as our author. But finding by experiment that the second was more conformable to nature, he afterwards made some progress in that, till he was stopt, by not being able to execute to his wish what related to the perpendicular descent of bodies; not observing that the measurement of the curve line, he made use of to explain it by, depended on the hyperbola. Which oversight may well be pardoned in that great man, considering that our author had not been pleased at that time to communicate to the publick his admirable discourse of the QUADRATURE OF MEASUREMENT OF CURVE LINES, with which he has since obliged the world: for without the use of that treatise, it is I think no injury even to our author's unparalleled abilities to believe, it would not have been easy for himself to have succeeded so happily in this and many other parts of his writings.

8. WHAT HUYGENS found by experiment, that bodies were in reality resisted in the duplicate proportion of their velocity, agrees with the reasoning of our author [b], who distinguishes the resistance, which fluids give to bodies by the tenacity of their parts, and the friction between them and the body, from that, which arises from the power of inactivity, with which the constituent particles of fluids are endued like all

[a] De la Pesanteur, pag. 169, and the following. | [b] Newton. Princ. L. II. prop. 4. schol.

 other

other portions of matter, by which power the particles of fluids like other bodies make reſiſtance againſt being put into motion.

9. THE reſiſtance, which ariſes from the friction of the body againſt the parts of the fluid, muſt be very inconſiderable; and the reſiſtance, which follows from the tenacity of the parts of fluids, is not uſually very great, and does not depend much upon the velocity of the body in the fluid; for as the parts of the fluid adhere together with a certain degree of force, the reſiſtance, which the body receives from thence, cannot much depend upon the velocity, with which the body moves; but like the power of gravity, its effect muſt be proportional to the time of its acting. This the reader may find farther explained by Sir ISAAC NEWTON himſelf in the poſtſcript to a diſcourſe publiſhed by me in THE PHILOSOPHICAL TRANSACTIONS, N° 371. The principal reſiſtance, which moſt fluids give to bodies, ariſes from the power of inactivity in the parts of the fluids, and this depends upon the velocity, with which the body moves, on a double account. In the firſt place, the quantity of the fluid moved out of place by the moving body in any determinate ſpace of time is proportional to the velocity, wherewith the body moves; and in the next place, the velocity with which each particle of the fluid is moved, will alſo be proportional to the velocity of the body: therefore ſince the reſiſtance, which any body makes againſt being put into motion, is proportional both to the quantity of matter moved and the velocity it is moved with; the reſiſtance, which a fluid gives on this account, will be doubly increaſed with the increaſe of the velocity in the moving body;
that

that is, the reſiſtance will be in a two-fold or duplicate proportion of the velocity, wherewith the body moves through the fluid.

10. FARTHER it is moſt manifeſt, that this latter kind of reſiſtance increaſing with the increaſe of velocity, even in a greater degree than the velocity it ſelf increaſes, the ſwifter the body moves, the leſs proportion the other ſpecies of reſiſtance will bear to this: nay that this part of the reſiſtance may be ſo much augmented by a due increaſe of velocity, till the former reſiſtances ſhall bear a leſs proportion to this, than any that might be aſſigned. And indeed experience ſhews, that no other reſiſtance, than what ariſes from the power of inactivity in the parts of the fluid, is of moment, when the body moves with any conſiderable ſwiftneſs.

11. THERE is beſides theſe yet another ſpecies of reſiſtance, found only in ſuch fluids, as, like our air, are elaſtic. Elaſticity belongs to no fluid known to us beſide the air. By this property any quantity of air may be contracted into a leſs ſpace by a forcible preſſure, and as ſoon as the compreſſing power is removed, it will ſpring out again to its former dimenſions. The air we breath is held to its preſent denſity by the weight of the air above us. And as this incumbent weight, by the motion of the winds, or other cauſes, is frequently varied (which appears by the barometer;) ſo when this weight is greateſt, we breath a more denſe air than at other times. To what degree the air would expand it ſelf by its ſpring, if all preſſure were removed, is not known,

known, nor yet into how narrow a compaſs it is capable of being compreſſed. Mr. BOYLE found it by experiment capable both of expanſion and compreſſion to ſuch a degree, that he could cauſe a quantity of air to expand it ſelf over a ſpace ſome hundred thouſand times greater, than the ſpace to which he could confine the ſame quantity [a]. But I ſhall treat more fully of this ſpring in the air hereafter [b]. I am now only to conſider what reſiſtance to the motion of bodies ariſes from it.

12. BUT before our author ſhews in what manner this cauſe of reſiſtance operates, he propoſes a method, by which fluids may be rendered elaſtic, demonſtrating that if their particles be provided with a power of repelling each other, which ſhall exert it ſelf with degrees of ſtrength reciprocally proportional to the diſtances between the centers of the particles; that then ſuch fluids will obſerve the ſame rule in being compreſſed, as our air does, which is this, that the ſpace, into which it yields upon compreſſion, is reciprocally proportional to the compreſſing weight [c]. The term reciprocally proportional has been explained above [d]. And if the centrifugal force of the particles acted by other laws, ſuch fluids would yield in a different manner to compreſſion [e].

13. WHETHER the particles of the air be endued with ſuch a power, by which they can act upon each other out of contact, our author does not determine; but leaves that

[a] See his Tract on the admirable rarifaction of the air.
[b] Book II. Ch. 6.
[c] Princ. philoſ. Lib. II. prop. 23.
[d] Book I. Ch. 2. § 30.
[e] Princ. philoſ. Lib. II. prop. 23. in ſchol.

3 to

to future examination, and to be discussed by philosophers. Only he takes occasion from hence to consider the resistance in elastic fluids, under this notion; making remarks, as he passes along, upon the differences, which will arise, if their elasticity be derived from any other fountain [a]. And this, I think, must be confessed to be done by him with great judgment; for this is far the most reasonable account, which has been given of this surprizing power; as must without doubt be freely acknowledged by any one, who in the least considers the insufficiency of all the other conjectures, which have been framed; and also how little reason there is to deny to bodies other powers, by which they may act upon each other at a distance, as well as that of gravity; which we shall hereafter shew to be a property universally belonging to all the bodies of the universe, and to all their parts [b]. Nay we actually find in the loadstone a very apparent repelling, as well as an attractive power. But of this more in the conclusion of this discourse.

14. BY these steps our author leads the way to explain the resistance, which the air and such like fluids will give to bodies by their elasticity; which resistance he explains thus. If the elastic power of the fluid were to be varied so, as to be always in the duplicate proportion of the velocity of the resisted body, it is shewn that then the resistance, derived from the elasticity, would increase in the duplicate proportion of the velocity; in so much that the

[a] Princ. philos. Lib. II. prop. 33. coroll. | [b] Lib. II. Ch. 5.

whole

whole resistance would be in that proportion, excepting only that small part, which arises from the friction between the body and the parts of the fluid. From whence it follows, that because the elastic power of the same fluid does in truth continue the same, if the velocity of the moving body be diminished, the resistance from the elasticity, and therefore the whole resistance, will decrease in a less proportion, than the duplicate of the velocity; and if the velocity be increased, the resistance from the elasticity will increase in a less proportion, than the duplicate of the velocity, that is in a less proportion, than the resistance made by the power of inactivity of the parts of the fluid. And from this foundation is raised the proof of a property of this resistance, given by the elasticity in common with the others from the tenacity and friction of the parts of the fluid; that the velocity may be increased, till this resistance from the fluid's elasticity shall bear no considerable proportion to that, which is produced by the power of inactivity thereof [a]. From whence our author draws this conclusion; that the resistance of a body, which moves very swiftly in an elastic fluid, is near the same, as if the fluid were not elastic; provided the elasticity arises from the centrifugal power of the parts of the medium, as before explained, especially if the velocity be so great, that this centrifugal power shall want time to exert it self [b]. But it is to be observed, that in the proof of all this our author proceeds upon the supposition of this centrifugal power in the parts of the fluid; but if the elasticity be caused by the expansion of the parts in the

[a] L. 2. Prop. 33. coroll. 2. | [b] Ibid. coroll. 3.

manner

manner of wool compreſſed, and ſuch like bodies, by which the parts of the fluid will be in ſome meaſure entangled together, and their motion be obſtructed, the fluid will be in a manner tenacious, and give a reſiſtance upon that account over and above what depends upon its elaſticity only [a]; and the reſiſtance derived from that cauſe is to be judged of in the manner before ſet down.

15. IT is now time to paſs to the ſecond part of this theory; which is to aſſign the meaſure of reſiſtance, according to the proportion between the denſity of the body and the denſity of the fluid. What is here to be underſtood by the word denſity has been explained above [b]. For this purpoſe as our author before conſidered two diſtinct caſes of bodies moving in mediums; one when they oppoſed themſelves to the fluid by their power of inactivity only, and another when by aſcending or deſcending their weight was combined with that other power: ſo likewiſe, the fluids themſelves are to be regarded under a double capacity; either as having their parts at reſt, and diſpoſed freely without reſtraint, or as being compreſſed together by their own weight, or any other cauſe.

16. IN the firſt caſe, if the parts of the fluid be wholly diſingaged from one another, ſo that each particle is at liberty to move all ways without any impediment, it is ſhewn, that if a globe move in ſuch a fluid, and the globe and par-

[a] Vid. ibid. coroll. 6. | [b] In § 2.

ticles of the fluid are endued with perfect elasticity; so that as the globe impinges upon the particles of it, they shall bound off and separate themselves from the globe, with the same velocity, with which the globe strikes upon them; then the resistance, which the globe moving with any known velocity suffers, is to be thus determined. From the velocity of the globe, the time, wherein it would move over two third parts of its own diameter with that velocity, will be known. And such proportion as the density of the fluid bears to the density of the globe, the same the resistance given to the globe will bear to the force, which acting, like the power of gravity, on the globe without intermission during the space of time now mentioned, would generate in the globe the same degre of motion, as that wherewith it moves in the fluid [a]. But if neither the globe nor the particles of the fluid be elastic, so that the particles, when the globe strikes against them, do not rebound from it, then the resistance will be but half so much [b]. Again, if the particles of the fluid and the globe are imperfectly elastic, so that the particles will spring from the globe with part only of that velocity wherewith the globe impinges upon them; then the resistance will be a mean between the two preceding cases, approaching nearer to the first or second, according as the elasticity is more or less [c].

17. The elasticity, which is here ascribed to the particles of the fluid, is not that power of repelling one another,

[a] Princ. philos. Lib. II. Prop. 35.
[b] Ibid.
[c] Id.

when out of contact; by which, as has before been mentioned, the whole fluid may be rendred elaſtic; but ſuch an elaſticity only, as many ſolid bodies have of recovering their figure, whenever any forcible change is made in it, by the impulſe of another body or otherwiſe. Which elaſticity has been explained above at large [a].

18. THIS is the caſe of diſcontinued fluids, where the body, by preſſing againſt their particles, drives them before itſelf, while the ſpace behind the body is left empty. But in fluids which are compreſſed, ſo that the parts of them removed out of place by the body reſiſted immediately retire behind the body, and fill that ſpace, which in the other caſe is left vacant, the reſiſtance is ſtill leſs; for a globe in ſuch a fluid which ſhall be free from all elaſticity, will be reſiſted but half as much as the leaſt reſiſtance in the former caſe [b]. But by elaſticity I now mean that power, which renders the whole fluid ſo; of which if the compreſſed fluid be poſſeſſed, in the manner of the air, then the reſiſtance will be greater than by the foregoing rule; for the fluid being capable in ſome degree of condenſation, it will reſemble ſo far the caſe of uncompreſſed fluids [c]. But, as has been before related, this difference is moſt conſiderable in ſlow motions.

19. IN the next place our author is particular in determining the degrees of reſiſtance accompanying bodies of different figures; which is the laſt of the three heads, we

[a] h. 1. § 29.
[b] Princ. philoſ. Lib. II. Prop. 38, compared with coroll. 1. of prop. 35.
[c] L. II. Lem. 7. ſchol. pag. 341.

divided the whole discourse of resistance into. And in this disquisition he finds a very surprizing and unthought of difference, between free and compressed fluids. He proves, that in the former kind, a globe suffers but half the resistance, which the cylinder, that circumscribes the globe, will do, if it move in the direction of its axis [a]. But in the latter he proves, that the globe and cylinder are resisted alike [b]. And in general, that let the shape of bodies be ever so different, yet if the greatest sections of the bodies perpendicular to the axis of their motion be equal, the bodies will be resisted equally [c].

20. Pursuant to the difference found between the resistance of the globe and cylinder in rare and uncompressed fluids, our author gives us the result of some other inquiries of the same nature. Thus of all the frustums of a cone, that can be described upon the same base and with the same altitude, he shews how to find that, which of all others will be the least resisted, when moving in the direction of its axis [d]. And from hence he draws an easy method of altering the figure of any spheroidical solid, so that its capacity may be enlarged, and yet the resistance of it diminished [e]: a note which he thinks may not be useless to shipwrights. He concludes with determining the solid, which will be resisted the least that is possible, in these discontinued fluids [f].

[a] Lib. II. Prop. 34.
[b] Lib. II. Lem. 7. p. 341.
[c] Schol. to Lem. 7.
[d] Prop. 34. schol.
[e] Ibid.
[f] Ibid.

21. That

21. THAT I may here be underſtood by readers unacquainted with mathematical terms, I ſhall explain what I mean by a fruſtum of a cone, and a ſpheroidical ſolid. A cone has been defined above. A fruſtum is what remains, when part of the cone next the vertex is cut away by a ſection parallel to the baſe of the cone, as in fig. 86. A ſpheroid is produced from an ellipſis, as a ſphere or globe is made from a circle. If a circle turn round on its diameter, it deſcribes by its motion a ſphere; ſo if an ellipſis (which figure has been defined above, and will be more fully explained hereafter [a]) be turned round either upon the longeſt or ſhorteſt line, that can be drawn through the middle of it, there will be deſcribed a kind of oblong or flat ſphere; as in fig. 87. Both theſe figures are called ſpheroids, and any ſolid reſembling theſe I here call ſpheroidical.

22. IF it ſhould be asked, how the method of altering ſpheroidical bodies, here mentioned, can contribute to the facilitating a ſhip's motion, when I juſt above affirmed, that the figure of bodies, which move in a compreſſed fluid not elaſtic, has no relation to the augmentation or diminution of the reſiſtance; the reply is, that what was there ſpoken relates to bodies deep immerged into ſuch fluids, but not of thoſe, which ſwim upon the ſurface of them; for in this latter caſe the fluid, by the appulſe of the anterior parts of the body, is raiſed above the level of the ſurface, and behind the body is ſunk ſomewhat below; ſo

[a] Book II. Ch. 1. § 6.

that

that by this inequality in the ſuperficies of the fluid, that part of it, which at the head of the body is higher than the fluid behind, will reſiſt in ſome meaſure after the manner of diſcontinued fluids [a], analogous to what was before obſerved to happen in the air through its elaſticity, though the body be ſurrounded on every ſide by it [b]. And as far as the power of theſe cauſes extends, the figure of the moving body affects its reſiſtance; for it is evident, that the figure, which preſſes leaſt directly againſt the parts of the fluid, and ſo raiſes leaſt the ſurface of a fluid not elaſtic, and leaſt compreſſes one that is elaſtic, will be leaſt reſiſted.

23. THE way of collecting the difference of the reſiſtance in rare fluids, which ariſes from the diverſity of figure, is by conſidering the different effect of the particles of the fluid upon the body moving againſt them, according to the different obliquity of the ſeveral parts of the body upon which they reſpectively ſtrike; as it is known, that any body impinging againſt a plane obliquely, ſtrikes with a leſs force, than if it fell upon it perpendicularly; and the greater the obliquity is, the weaker is the force. And it is the ſame thing, if the body be at reſt, and the plane move againſt it [c].

24. THAT there is no connexion between the figure of a body and its reſiſtance in compreſſed fluids, is proved thus. Suppoſe ABCD (in fig. 88.) to be a canal, having ſuch a fluid, water for inſtance, running through it with an equable

[a] Vid. Newt. princ. in ſchol. to Lem. 7, of Lib. II. pag. 341.

[b] Sect. 17. of this chapter.

[c] See Princ. philoſ. Lib. II. prop. 34.

velocity;

velocity; and let any body E, by being placed in the axis of the canal, hinder the paſſage of the water. It is evident, that the figure of the fore part of this body will have little influence in obſtructing the water's motion, but the whole impediment will ariſe from the ſpace taken up by the body, by which it diminiſhes the bore of the canal, and ſtraightens the paſſage of the water [a]. But proportional to the obſtruction of the water's motion, will be the force of the water upon the body E [b]. Now ſuppoſe both orifices of the canal to be cloſed, and the water in it to remain at reſt; the body E to move, ſo that the parts of the water may paſs by it with the ſame degree of velocity, as they did before; it is beyond contradiction, that the preſſure of the water upon the body, that is, the reſiſtance it gives to its motion, will remain the ſame; and therefore will have little connexion with the figure of the body [c].

25. BY a method of reaſoning drawn from the ſame fountain is determined the meaſure of reſiſtance theſe compreſſed fluids give to bodies, in reference to the proportion between the denſity of the body and that of the fluid. This ſhall be explained particularly in my comment on Sir IS. NEWTON's mathematical principles of natural philoſophy; but is not a proper ſubject to be inſiſted on farther in this place.

26. WE have now gone through all the parts of this theory. There remains nothing more, but in few words to mention the experiments, which our author has made, both

[a] Vid. Princ. philoſ. Lib. II. Lem. 5. p. 314. | [b] Lemm. 6. | [c] Ibid. 7.

with

with bodies falling perpendicularly through water, and the air [a], and with pendulums [b]: all which agree with the theory. In the caſe of falling bodies, the times of their fall determined by the theory come out the ſame, as by obſervation, to a ſurprizing exactneſs; in the pendulums, the rod, by which the ball of the pendulum hangs, ſuffers reſiſtance as well as the ball, and the motion of the ball being reciprocal, it communicates ſuch a motion to the fluid, as increaſes the reſiſtance; but the deviation from the theory is no more, than what may reaſonably follow from theſe cauſes.

27. By this theory of the reſiſtance of fluids, and theſe experiments, our author decides the queſtion ſo long agitated among natural philoſophers, whether all ſpace is abſolutely full of matter. The Ariſtotelians and Carteſians both aſſert this plenitude; the Atomiſts have maintained the contrary. Our author has choſe to determine this queſtion by his theory of reſiſtance, as ſhall be explained in the following chapter.

[a] Newt. Princ. Lib. II. prop. 40. in ſchol.

[b] Lib. II. in ſchol. poſt prop. 31.

BOOK